

ASTRON.
CBS.
VOLUME LIII

NUMBER 3

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE

Mount Wilson Observatory of the Carnegie
Institution of Washington

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APRIL 1921

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THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS, U.S.A.

THE CAMBRIDGE UNIVERSITY PRESS, London
THE MANUZEN-KABUSHIKI-KAISHA, Tokyo, Osaka, Kyoto, Fukuoka, Sendai
THE MISSION BOOK COMPANY, Shanghai

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The *Astrophysical Journal* is published by the University of Chicago at the University of Chicago Press, 5750 Ellis Avenue, Chicago, Illinois, during each month except February and August. ¶The subscription price is \$6.00 a year; the price of single copies is 75 cents. Orders for service of less than a half-year will be charged at the single-copy rate. ¶Postage is prepaid by the publishers on all orders from the United States, Mexico, Cuba, Porto Rico, Panama Canal Zone, Republic of Panama, Bolivia, Colombia, Honduras, Nicaragua, Peru, Hawaiian Islands, Philippine Islands, Guam, Samoan Islands, Shanghai. ¶Postage is charged extra as follows: For Canada, 30 cents on annual subscriptions (total \$6.30), on single copies, 3 cents (total 78 cents); for all other countries in the Postal Union, 62 cents on annual subscriptions (total \$6.62), on single copies, 11 cents (total 86 cents). ¶Patrons are requested to make all remittances payable to The University of Chicago Press in postal or express money orders or bank drafts.

The following are authorized to quote the prices indicated:

For the British Empire: The Cambridge University Press, Fetter Lane, London, E.C. 4. Yearly subscriptions, including postage, 43s. each; single copies, including postage, 5s. 6d. each.

For Japan and Korea: The Maruzen-Kabushiki-Kaisha, 11 to 16 Nihonbashi Tori Sanchoe, Tokyo, Japan. Yearly subscriptions, including postage, Yen 13.30 each; single copies, including postage, Yen 1.75 each.

For China: The Mission Book Company, 13 North Szechuen Road, Shanghai. Yearly subscriptions, \$6.00; single copies, 75 cents, or their equivalents in Chinese money. Postage extra, if mailed direct outside of Shanghai, on yearly subscriptions 62 cents, on single copies 11 cents.

Claims for missing numbers should be made within the month following the regular month of publication. The publishers expect to supply missing numbers free only when losses have been sustained in transit, and when the reserve stock will permit.

Business correspondence should be addressed to The University of Chicago Press, Chicago, Illinois.

Communications for the editors and manuscripts should be addressed to the Editors of THE ASTROPHYSICAL JOURNAL, Yerkes Observatory, Williams Bay, Wisconsin.

The cable address is "University, Chicago."

The articles in this Journal are indexed in the *Readers' Guide Supplement*, New York, N.Y.

Entered as second-class matter, January 17, 1895, at the Post-office at Chicago, Ill., under the act of March 3, 1879.

Acceptance for mailing at special rate of postage provided for in Section 3709, Act of October 3, 1917, authorized on July 15, 1918.

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ON THE PASSAGE OF A STAR THROUGH A NEBULA

By ERNEST W. BROWN

ABSTRACT

Effect of the passage of a star through a nebula consisting of a swarm of particles; theory.—Neglecting the effects of mutual collisions between the particles when the swarm is undisturbed as well as the gravitational attraction of the swarm on its constituents, the author derives the formulae for the hyperbolic motion of the particles and shows that all those originally at a distance b from the path of the star will cross that path at the same point at a distance $b^2/2a$ behind the star, where a is the ratio of the mass of the star to the square of its speed, V^2 . If we suppose that as a result of this focusing action collisions occur along the path of the star, these might produce a gaseous appendage of conical form with an angle β such that $\sin \beta$ is equal to the ratio of the mean velocity of the gas molecules to V .

Variable nebula N.G.C. 2261 (Hubble), with fan-shaped appendage.—The above theory suggests an *explanation* of the form of the appendage, and certain numerical consequences as to the mass, scale of magnitude, distance, and life of this object are deduced.

The general problem of the phenomena produced when a star encounters a nebula is one of great complexity and one which can only be properly treated by extended mathematical and physical investigations. In any case it must demand numerous assumptions concerning the physical conditions of both star and nebula. But there are certain forms of the problem which will yield to more simple treatment and which may furnish a qualitative idea of the phenomena and also some indications of the numerical magnitudes involved. It is one of them to which attention is drawn in this paper, illustrations being furnished by certain

planetary (?) nebulae and in particular by Hubble's¹ variable nebula, N.G.C. 2261 = H. IV 2 = h 399 = G.C. 1437.

I assume at the outset that the "nebula" involved is a swarm of discrete particles or small masses, the swarm being of very great extent but with such low volume density that we may neglect the effects of mutual collisions between the particles when the swarm is undisturbed, as well as the gravitational attraction of the swarm on its own constituents. The star is supposed to be passing through the interior of the swarm with constant velocity relative to the swarm: this involves the further assumption that the retardation of its motion produced by collisions is very small compared with the other velocities involved. Since the relative motion of the star and the swarm is alone involved in the investigation, I shall suppose that the star is at rest and that the particles of the swarm have a velocity V , constant in magnitude and direction at a great distance from the star, this velocity being only

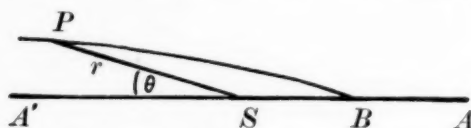


FIG. 1

changed on the approach of the particle to the star by the attraction of the latter.

Under these conditions, every particle of the swarm moves as though the rest of the swarm did not exist, except in so far as the attraction of the star may later cause collisions.

Let S be the center of the star and draw $A'SA$ parallel to the direction of motion of each particle of the swarm when at a great distance from the star. Let PB be the orbit of a particle during its passage near the star. This orbit is a hyperbola, the antecedent asymptote of which is parallel to $A'SA$; let the orbit intersect SA in B . Since we have to deal with a swarm, there will exist a ring of such orbits, all of them intersecting at B , which will therefore be a center of collision for the particles following these orbits. There will similarly be centers of collision at all points of SA ,

¹ *Astrophysical Journal*, 44, 190, 1916.

starting from some point near S and extending to infinity in the direction SA .

Three classes of phenomena have to be considered. First, the orbits of particles before they intersect the line SA ; second, the motions after collision along this line; third, the effects of particles which collide with the star, since we must suppose the latter to have an appreciable diameter.

Under the limitations imposed, the investigation of the first is simply that of hyperbolic motion applied to this particular case. The effect of the third may be dismissed briefly, since the supposition that the relative velocity of the star and of the main body of the swarm is not sensibly affected will be retained throughout. The second is that which presents the features of chief interest and which is assumed to be mainly responsible for the visible phenomena.

HYPERBOLIC MOTION

I now develop the formulae for hyperbolic motion which will be required.

Take S as origin and SA' the initial line for the polar co-ordinates r, θ of a particle P at time t . Let μ be the mass of the star in astronomical units and b the perpendicular distance of P from SA' at time $t = -\infty$, when it is moving parallel to $A'S$ with velocity V . Then the well-known equations of motion are

$$\left. \begin{aligned} \frac{d^2r}{dt^2} - r \left(\frac{d\theta}{dt} \right)^2 &= -\frac{\mu}{r^2}, \\ r^2 \frac{d\theta}{dt} &= \text{moment of velocity} = \text{constant} = bV \end{aligned} \right\}. \quad (1)$$

The equation of the conic which satisfies these equations is

$$\frac{1}{r} = A \cos \theta + B \sin \theta + C, \quad (2)$$

where A, B, C are constants to be determined by the given conditions of the problem.

Differentiating (2) with respect to the time and substituting for $d\theta/dt$ from the second of equations (1) we obtain

$$\frac{dr}{dt} = (A \sin \theta - B \cos \theta) bV. \quad (3)$$

Differentiating again and substituting for d^2r/dt^2 and for $d\theta/dt$ from (1) we find

$$(A \cos \theta + B \sin \theta) \frac{b^2 V^2}{r^2} - \frac{b^2 V^2}{r^3} = -\frac{\mu}{r^2},$$

or

$$A \cos \theta + B \sin \theta - \frac{1}{r} = -\frac{\mu}{b^2 V^2} = -\frac{a}{b^2}, \quad (4)$$

in which we have defined a by putting

$$\mu = a V^2.$$

To determine A , B , C we are given that $dr/dt = -V$ and $r = \infty$ when $\theta = 0$. Substituting these values in (2), (3), (4), we obtain

$$A + C = 0, \quad -V = -B \cdot b V, \quad A = -\frac{a}{b^2}.$$

Hence (2) and (3) become

$$\frac{b}{r} = (1 - \cos \theta) \frac{a}{b} + \sin \theta, \quad (2a)$$

$$\frac{dr}{dt} = -V \left(\cos \theta + \frac{a}{b} \sin \theta \right), \quad (3a)$$

to which the energy equation,

$$v^2 = \left(\frac{dr}{dt} \right)^2 + r^2 \left(\frac{d\theta}{dt} \right)^2 = \frac{2\mu}{r} + V^2, \quad (5)$$

may be joined, where v is the velocity at distance r .

Let SB be denoted by c . Then $r = c$ when $\theta = \pi$ and we obtain from (2a), (3a),

$$r = c = \frac{b^2}{2a}, \quad \frac{dr}{dt} = V, \quad \text{when } \theta = \pi. \quad (6)$$

The point of closest approach to the star (periastron) is obtained by putting $dr/dt = 0$; let $\theta = \alpha$ at this point. Then from (3a) we have $\tan \alpha = -b/a$. Thence from (2a),

$$r = \sqrt{a^2 + b^2} - a \quad \text{when } dr/dt = 0. \quad (7)$$

Let R be the effective radius of the star, that is, the closest distance at which any particle does not collide with any part of the star's mass. The smallest value of b for such a particle is,

from (7), given by $b^2 = R^2 + 2aR$. Since $c = b^2/2a$, the smallest possible value of c (which will be denoted by c_0) is given by the equation

$$c_0 = \frac{R^2}{2a} + R. \quad (8)$$

It is understood that when the star has a gaseous envelope any particle which is retarded by the envelope is retained by the star; this assumption can be modified later.

We now deal with the motions of the particles after the collisions have taken place along the line SA . The velocities perpendicular to SA being destroyed by the collisions, the particles, supposed inelastic, will begin to move along SA with velocity V , as shown by equation (6), whatever value c may have; this velocity will be decreased in the succeeding motion by the attraction of the star.

The ideal case in which all collisions take place on SA will be retained while the effect of the collisions is considered. I suppose that enough heat is generated by the collisions to turn all the incoming matter into a gaseous condition and that the gas immediately starts to obey the laws of an ordinary gas. Under the conditions laid down, there is no restriction on the motions of its molecules, that is, there is no external pressure. They will therefore begin to move with their proper molecular velocities relatively to the motion of the whole mass. Let the mean molecular relative velocity be U . From each point of SA will then start a spherical wave whose radius increases at the rate U while (if we neglect the retardation produced by the attraction of the star) the whole sphere moves with velocity V parallel to SA . The wave front will therefore lie along a cone of angle β such that

$$\sin \beta = \frac{U}{V}.$$

Molecular velocities are of the order of a kilometer per second while stellar velocities are of the order of twenty kilometers per second on the average so that we should expect the cone in general so have a very small angle. If, however, V be of the order of two kilometers per second, the angle rises to the order of 30° .

With a low relative velocity we cannot neglect the retardation produced by the attraction of the star. The principal effect is to diminish V as t increases and therefore to increase β . The generating lines of the cone enveloping the wave-front, instead of being straight, will be curved with convexity toward SA .

Next, the particles of the swarm are pouring into this expanding gas when the process is in full operation and the collisions no longer take place on SA ; they will in general be changed into a gaseous condition by collisions with the molecules of the latter before they reach SA . In the mean, the velocities perpendicular to SA are still destroyed owing to the symmetry of the whole process with respect to this line. The velocity parallel to SA before collision at any point whose co-ordinates are r, θ , is from (1), (3a), (2a),

$$\begin{aligned} \frac{dr}{dt} \cos \theta - r \frac{d\theta}{dt} \sin \theta &= -V \left(\cos^2 \theta + \frac{a}{b} \sin \theta \cos \theta \right) - \frac{bV}{r} \sin \theta \\ &= -V \left\{ \cos^2 \theta + \frac{a}{b} \sin \theta \cos \theta + \frac{a}{b} \sin \theta (1 - \cos \theta) + \sin^2 \theta \right\} \\ &= -V \left(1 + \frac{a}{b} \sin \theta \right). \end{aligned}$$

As θ is less than π for all such cases, the actual velocity parallel to SA communicated to the expanding gas is greater than if the collisions took place on SA , but tends toward V with increasing magnitude of the periastron distance. The tendency is therefore to diminish the angle of the cone. This action, however, is also less effective near the vertex of the cone; the result is to produce a curvature of the generating lines of the wave-front surface with concavity toward SA and therefore in the opposite direction to that found in the previous paragraph. Considerable internal motion will be produced in the expanding gas, but I am chiefly considering here the outline of the wave-front.

Next consider these portions of the stream which collide with the star so as to become portions of its mass. Whatever the original condition of the star on the approach of the swarm, the collisions will heat up its surface and in time produce a gaseous envelope which will increase the effective radius of the star, the

latter being the radius within which capture of the particles of the swarm takes place. Further, although the captures take place mainly in the front portion of the envelope, the impulses as well as the gravitational attraction of the nucleus will tend to spread the envelope to the rear portions so that the envelope will probably remain approximately spherical.

There will be a number of particles which are not sufficiently retarded by the outer portions of the envelope to be captured; these will pass through with velocity diminished by this cause and so intersect SA in points which are at a less distance from S than C_0 . A clear outline of the vertex of the fan cannot therefore be expected.

Before we attempt to compare these deductions with observation it is necessary to examine the light given out. Consider the mass of gas produced at any moment in the envelope of the star and in the fan before it has been cooled by expansion and radiation. The latter processes are gradual and considered alone would produce in the fan a triangular-shaped projection whose central portions are bright, with diminishing light toward the sides and rear. But the incoming stream on meeting the wave-front should be rapidly converted into heated gas and so tend to outline the latter with some approach to definiteness, and there does not seem to be any reason to expect systematic variations in the light of any part of the projection along lines perpendicular to SA ; naturally, toward the rear there will be a gradual diminution of the light given out. No such definite outline is to be expected in the envelope of the star since its matter is being continually arranged in layers of increasing density with but little loss of heat due to expansion; the interior portions will have the highest temperature.

This attempt to outline the character of the phenomena produced under a speculative hypothesis must be treated with considerable reserve in the absence of any calculation. But it does not seem useful at the outset to go into details before the possibilities and probabilities of the principal features have been under discussion. With the knowledge which has been gathered of the distribution and motion of matter in space it appears that the passage of a star through widely diffused matter will be by no

means a rare phenomenon like that of the close approach of two stars. As a matter of fact this investigation was begun in order to examine the possibilities it might furnish of the evolution of a planetary system. During a visit to Mount Wilson last spring in which the idea was discussed, my attention was called to Hubble's variable nebula as possibly furnishing an example of the phenomena outlined above. As is well known, this object consists mainly of a starlike nucleus (which is known as the variable star R Monocerotis) with a fan-shaped appendage. Attention, however, had been mainly directed to the variability in the light-giving power of the fan rather than to its mode of formation.

On the hypothesis of this paper the sine of half the angle formed by the two sides of the fan should give the ratio of U to V , that is, the ratio of the molecular velocity of the expanding gas to the relative velocity of the star and swarm. In Hubble's nebula (N.G.C. 2261) this half-angle is about 30° , giving a relative velocity of the order of two kilometers per second. The equation $\mu = aV^2$ thence gives a relation between μ the mass of the star and a the "scale" of the system. Let μ_0 be the mass of the sun, V_0 the mean orbital velocity of the earth, and a_0 the mean radius of the earth's orbit. Then since V_0 is approximately 30 kilometers per second, we have

$$\frac{\mu}{\mu_0} = \frac{a}{a_0} \frac{V^2}{V_0^2} = \frac{1}{225} \frac{a}{a_0}. \quad (9)$$

By equation (8) the distance of the vertex of the cone from the center of the star is

$$c_0 = \frac{R^2}{2a} + R. \quad (8)$$

The photographs seem to show that c_0 and R are of the same order of magnitude. If $c_0 = 2R$, we have $a = \frac{1}{2}R$. It is to be noted, however, that a increases rapidly with a decrease of $c_0 - R$. It is immaterial which of these quantities we use to denote the scale of magnitude of the system. I have adopted a for convenience.

There is no direct information available as to the parallax, but from indirect evidence Hubble (*loc. cit.*) believes that it is measurable, provided suitable definition of the center of the nucleus

can be obtained photographically. The general information now available concerning the magnitudes of such objects seems to indicate the same fact. We may, in fact, expect a mass comparable with that of a star. If this is the case we should expect a corresponding scale of linear magnitude. If the mass of the nucleus of the nebula is the same as that of the sun, equations (8), (9) show that the effective radius would be 450 times that of the earth's orbit. It may be conjectured that the mass lies between $1/10$ and 10 times that of the sun, giving an effective radius between 45 and 4500 times the radius of the earth's orbit. With the exposures hitherto made¹ the maximum radius of the envelope is about $5''$. If this be also the effective radius, the parallax should lie between $0''.1$ and $0''.001$, so that there is reason to hope that a direct measure of this latter quantity may be obtainable.

The nebula has been photographed for about twenty years. In the period 1908 to 1916, there is no very noticeable change in the angle of the fan, indicating that, on the hypothesis of this paper, the change in relative velocity during this time is small. The same thing is indicated in the later photographs of Lampland. If we suppose that the velocity during 20 years has averaged 2 kilometers per second, the distance traversed in that time will be 1.3×10^9 kilometers, a small quantity in comparison with the extent of nebular clouds. There seems to be no reason why the present conditions may not continue for a considerable period.

The outstanding feature of the fan is its rapid, irregular variation in light-giving power in different parts while its outline remains approximately the same. These variations may be attributed to local variations of density in the swarm and perhaps also to cumulative effects in the interior of the fan which are released in pulses rather than continuously. It is premature to discuss hypotheses to account for the fact that one side of the projected fan is slightly concave to the interior while the other side is slightly convex. There is a small circular condensation close to the nucleus and in front of it at an angle of about 45° to the axis of the fan. Hubble (*loc. cit.*) believes that there is evidence that this object has moved

¹ I am indebted to Mr. C. O. Lampland for the opportunity to examine many of the photographs which he has taken of this nebula at the Lowell Observatory.

toward the center of the nucleus at the rate of about $0''.5$ a year: if motion of this object around the nucleus can be detected, some interesting further possibilities in the development of the hypothesis of this paper will be open to consideration. It is, however, difficult to see any explanation for two faint wisps of nebulosity projected to some distance from the nucleus in front and symmetrically situated with respect to the axis of the fan, although they appear to follow the tracks of particles which later collide with the star.

In No. 81 of the *Bulletin of the Lowell Observatory* (3, 63, 1918) V. M. Slipher states that the nucleus and the fan have the same spectrum. It is necessary for the hypothesis that this should be so, for the external part of the nucleus as well as the fan must consist mainly of gas derived from the particles of the swarm after the envelope has once been formed. Slipher and Lampland also notice that the spectrum resembles that of a nova in its earlier stages, an interesting observation in view of the speculations which have been made as to the mode of formation of the latter objects.

Certain other nebulae exist in which the fan-shaped appendage is replaced by one with nearly parallel sides. Opportunity to examine photographs of these was also afforded me at Flagstaff last spring. If these nebulae are formed in a manner similar to that assumed for Hubble's nebula, we should conclude that the relative velocity of the swarm and star was much greater than 2 kilometers per second. In view of known stellar velocities these should prove to be the more common objects of this class. Further, it is to be noted that these objects must be seen in a direction nearly perpendicular to that of relative motion. For such objects viewed along or nearly along this direction, the appearance would be very different, ranging from an oval-shaped nebulosity with the star on the longer axis but away from the center, to a circular nebulosity surrounding the star. It might be hoped that relative displacements along the line of sight could be measured as a test, but the faintness of these objects is at present a serious obstacle to such measures. References to previous observations of these objects will be found in the first paper of Hubble referred to above.

YALE UNIVERSITY

December 1920

ON THE GEOMETRICAL CLASSIFICATION OF LONG-PERIOD VARIABLES

By J. G. HAGEN

ABSTRACT

Classification of long-period variables.—The writer gives a table of the *constants of the light-curves* of 66 variables as determined by harmonic analysis, and shows that on the basis of these constants the variables fall into three groups which coincide with the three main classes given in the *Harvard Annals* for 1907: (1) with uniform variation, (2) with broad minima, and (3) with a rapid increase. The first group corresponds in general to Phillips' Group I, and the last two are subdivisions of his Group II.

I

A large amount of material on long-period variable stars is contained in the *Harvard Annals*, 57, Part 1 (1907). The observations were mostly made at Harvard and the reduction was intrusted to Mr. L. Campbell. The results are there presented in very convenient form, in a statistical Table XII and on two Plates I and II.

1. Of all the seventy-five stars discussed in that volume, sixty-seven were found regular enough for showing graphically the different forms of light-curves. They are divided into five groups, two of which comprise only four stars each and might well be taken as modified forms of one or other of the larger groups. In fact the four curves of "broad maxima" and the other four of "rapid decrease" look like accidental irregularities from the large class of "uniform variation." Thus premised, the two Harvard plates exhibit three classes of light-curves with these characteristics: uniform variation, broad minima, rapid increase.

2. It is true that on page 209 it is expressly stated that these groups must not be considered as establishing different classes of long-period variables, and it may be that no further attention would have been paid to them, had not the Rev. T. E. R. Phillips subjected the very same stars to harmonic analysis and, in the main, reproduced the same classification. In his presidential

address before the British Astronomical Association, October 25, 1916, Phillips gave the results of his analysis and, from the relation between the phases of the second and third harmonics, recognized two principal groups of light-curves.

3. The writer has followed up the subject in the *Monthly Notices* (79, 572, 1919) and in the *Astronomische Nachrichten* (209, 257, 1919) and tried to show that from the same harmonics three groups of light-curves can be deduced and that these groups coincide with the three classes of the Harvard plates. The coincidence was naturally to be expected because the coefficients and phases of the harmonic formulae are deduced from the ordinates of the light-curves and cannot reveal qualities other than those contained in the drawings. Yet the writer's argument did not seem generally convincing, probably because he did not enumerate the stars of each group and did not represent the groups in a diagram. Supplying both points seems to be justified by the importance of the subject. While the short-period variables are divided into two classes distinct not only apparently by the shape of their light-curves but in their physical constitution, the long-period variables are so far from being physically understood, that at least their geometrical classification should be brought out as clearly as possible.

II

The following three tables contain all the stars whose light-curves are graphically represented on the two Harvard plates, though they do not comprise all the stars examined by Phillips. The stars are arranged by Pickering's catalogue numbers, i.e., in the order of R.A.

1. In the column $(M-m)/P$ the notation is Chandler's for the time from minimum to the next maximum divided by P , the period of variation. The fraction is readily transformed into the quantity considered by Professor Turner:

$$a = \{2(M-m) - P\}/P.$$

The last three columns are taken from Tables II and III of Phillips, including also the doubtful stars, which are distinguished

by parentheses. It appears at once that Phillips examined all the Harvard stars except V Cancri.

The angles ϕ_2 and ϕ_3 are the phases of the second and third harmonics in the formulae of the light-curves; wherever necessary they have been diminished by 360° in order to make their differences

TABLE I
STARS OF UNIFORM VARIATION

Star's No.	Name	$(M-m)/P$	ϕ_1	ϕ_2	Phillips
001726.....	T Andromedae	0.47	+110°	+80°	II
001755.....	T Cassiopeiae	.58	-36	191	I
011272.....	S Cassiopeiae	.42	+135	107	II
021024.....	R Arietis	.50	+13	107	I
050953.....	R Aurigae	.53	-58	216	I
065355.....	R Lyncis	.46	+42	211	I
072708.....	S Canis Minoris	.48	+33	207	I
084803.....	S Hydrae	.47	-24	189	I
085008.....	T Hydrae	.49	+(10)	(360)
094211.....	R Leonis	.45	+114	154	I
123307.....	R Virginis	.48	+10	216	I
123961.....	S Ursae Majoris	.48	+7	263	I
124606.....	U Virginis	.49	-15	197	I
132202.....	V Virginis	.46	+179	182	II
134440.....	R Canum Ven.	.49	+14	205	I
141954.....	S Boötis	.49	-2	258	I
142584.....	R Camelopardalis	.50	+7	213	I
143227.....	R Boötis	.48	+126	202	I
151520.....	S Librae	.37	+37	186	I
163266.....	R Draconis	.44	+122	60	II
164715.....	S Herculis	.45	-77	187	I
170215.....	R Ophiuchi	.47	+192	195	II
180531.....	T Herculis	.48	+51	160	I
191019.....	R Sagittarii	.44	+147	94	II
201008.....	R Delphini	.49	+166	171	II
201647.....	U Cygni	.44	+28	188	I
204405.....	T Aquarii	.50	+(171)	(242)
205023.....	R Vulpeculae	.41	+43	153	I
210868.....	T Cephei	.54	-65	205	I
213678.....	S Cephei	0.59	-77	+204	I
Sum.....	30	14.34	1403	5693
Mean.....	0.48	+47°	+190°

from the mean of each group less than 180° . The angles thus reduced will give more correct figures for the "range" within which they vary for each group than the uncorrected angles.

In the last column the "group" found by Phillips is assigned to each star.

TABLE II
STARS WITH BROAD MINIMA

Star's No.	Name	$(M-m)/P$	ϕ_1	ϕ_2	Phillips
001909.....	S Ceti	0.46	+178°	+195°	II
012502.....	R Piscium	.45	154	113	II
021403.....	o Ceti (Mira)	.36	118	59	II
042209.....	R Tauri	.40	180	107	II
042309.....	S Tauri	.26	149	105	II
081112.....	R Cancrī	.52	169	154	II
120905.....	T Virginis	.37	177	159	II
121418.....	R Corvi	.45	162	147	II
122803.....	Y Virginis	.44	(154)	(28)
132422.....	R Hydrae	.43	165	181	II
132706.....	S Virginis	.46	159	161	II
151714.....	S Serpentis	.43	176	125	II
161122 <i>b</i>	S Scorpii	.54	167	140	II
162119.....	U Herculis	.40	154	111	II
163137.....	W Herculis	.43	148	125	II
200357.....	S Cygni	.52	233	+206	I
203847.....	V Cygni	.38	(138)	-(26)
230110.....	R Pegasi	.35	164	+133	II
233815.....	R Aquarii	0.43	+176	+160	II
Sum.....	19	8.08	3121	2383
Mean.....	0.43	+164°	+125°

TABLE III
STARS WITH RAPID INCREASE

Star's No.	Name	$(M-m)/P$	ϕ_1	ϕ_2	Phillips
001838.....	R Andromedae	0.38	+ 97°	+ 66°	II
022000.....	R Ceti	.37	80	226	I
022813.....	U Ceti	.31	55	241	I
054920.....	U Orionis	.39	126	106	II
070122 <i>a</i>	R Geminorum	.42	98	35	II
081617.....	V Cancrī	.42
093934.....	R Leonis Minoris	.40	141	106	II
103769.....	R Ursae Majoris	.46	126	75	II
115919.....	R Comae Ber.	.33	122	+ 33	II
123160.....	T Ursae Majoris	.42	94	- 4	II
151731.....	S Coronae	.40	104	+ 69	II
160118.....	R Herculis	.44	136	102	II
161122 <i>a</i>	R Scorpii	.37	136	137	II
193449.....	R Cygni	.36	103	93	II
194632.....	χ Cygni	.46	(131)	(154)
225120.....	S Aquarii	.36	103	21	II
231508.....	S Pegasi	.43	105	93	II
235350.....	R Cassiopeiae	0.40	+118	+ 89	II
Sum.....	17 (+V Cancrī)	7.12	1875	1642
Mean.....	0.40	+110°	+ 97°

2. There are some discrepancies between the two classifications. Among the thirty stars of uniform variation seven are ascribed to Group II, but three of them are near the intersection of the two straight lines in Phillips' diagram. Again there is one star of Group I among the nineteen stars with broad minima, also near the point of intersection, and two stars of Group I in the third table. A percentage of six contradictory cases among sixty-six common stars seems to be small, in view of the fact that partly different light-curves were used for the two classifications. How much the light-curves of long-period variables differ from one appearance of the star to another may be seen from the monographs of Heis, Guthnick, and Rosenberg on the typical stars α Ceti and χ Cygni.

TABLE IV
MEAN RESULTS

GROUPS			LIGHT-CURVE		CO-ORDINATES		RANGE	
Phillips	Harvard	Stars	$(M-m)/P$	α	ϕ_2	ϕ_3	ϕ_2	ϕ_3
I.	Uniform variation.	30	0.48	-0.04	+ 47°	+ 190°	206°	300°
IIa. ...	Broad minimum...	19	0.43	-0.15	+ 164	+ 125	115	212
IIb. ...	Rapid increase.	17	0.40	-0.21	+ 110	+ 97	110	245

III

The discussion of the results is made easy by Table IV in which the mean values of the preceding tables are united, and by the accompanying diagram, where the phase-angles ϕ_2 and ϕ_3 are taken as co-ordinates of the stars, following Phillips' model except that different types are used to distinguish the stars of the three groups. It is to be noted that the diagram has eight stars less than that of Phillips.

1. The main result brought out clearly by both table and diagram consists in this, that there are three characteristic groups of light-curves and that these groups are roughly identical with the three classes of the Harvard plates: curves of uniform variation, curves with broad minima, and curves with rapid increase.

2. A secondary feature of Table IV and of the diagram is that uniform variation appears to be the general rule and that the other two types of light-curves are more like exceptions, though rather frequent ones. Indeed, several columns of Table IV show

a numerical progression from the first to the third class, e.g., in the values of $(M-m)/P$ or their equivalent values a , so also does the column of the mean co-ordinates ϕ_3 . Yet on the whole the first group differs more from the other two than these differ among themselves. The first group comprises nearly one-half of all the stars examined and the range of its phase-angles is much wider than that of the other groups. For this reason the

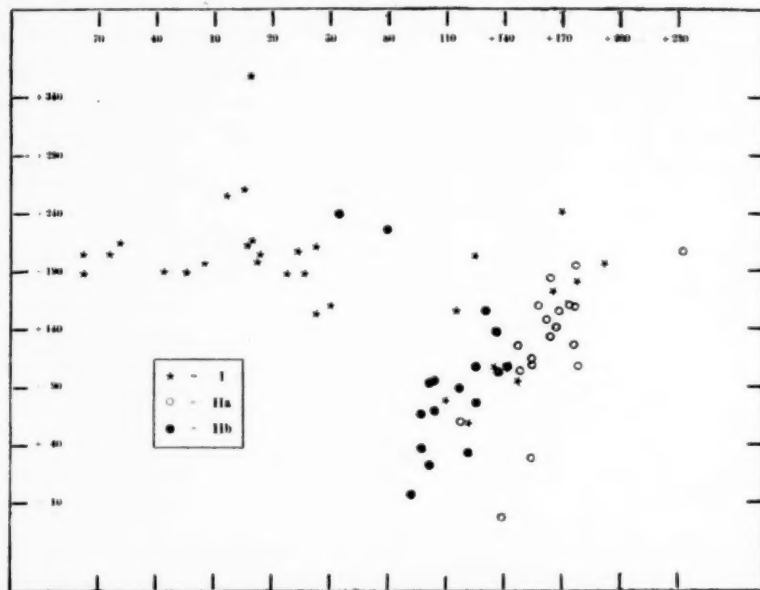


FIG. 1.—Classification of long-period variables. Ordinates, -10 to $+340$, represent ϕ_3 ; abscissae, -70 to $+230$, represent ϕ_2 .

main division, proposed by Phillips, into Groups I and II is justified: it separates the uniform light-curves from the less regular ones; but these deviations from the regular type should be specified by the subgroups IIa and IIb.

By thus giving the classification of the long-period variables its precise geometrical interpretation it is hoped that a small step is made forward toward the physical explanation of these stars, a problem for which no definite solution has so far been obtained.

VATICAN OBSERVATORY, ROME
September 12, 1920

CHARACTERISTIC BEHAVIOR OF THE BRIGHT LINES IN STELLAR SPECTRA OF CLASS Md¹

By PAUL W. MERRILL

ABSTRACT

Periodic changes in the emission lines of spectra of Class Md variables.—R Leonis, R Hydrae, R Serpentis, X Ophiuchi, χ Cygni, and T Cephei have been observed spectroscopically over a considerable part of their light periods, from two to twelve spectrograms having been made of each. The intensities of Hδ, Hγ, λ 4202 Fe, λ 4308 Fe, λ 4571 Mg, and some other bright lines at the different phases are recorded in Table IV. A compilation and discussion of past and present data referring to these lines shows that they appear after light minimum, in the order mentioned, in every long-period variable of Class Md whose spectrum has been adequately observed, and exhibit substantially the same remarkable behavior as in the type star α Ceti. There is strong evidence, therefore, that these spectral variations, shown in Figure 1, are characteristic of stars of this class. The changes in relative intensity of the lines of the Balmer series of hydrogen and also of certain iron lines are particularly interesting. A discussion of the physical interpretation of these variations is reserved until more data are available.

Although it has been known for many years, chiefly through the extensive labors of the Harvard College Observatory, that striking changes occur in the spectra of numerous long-period variable stars, practical difficulties have prevented the securing of satisfactory observations through the whole light cycle. It is therefore not yet possible to give a complete description of the spectroscopic changes which accompany the extraordinary fluctuations in brightness. Much valuable material has, however, already been gathered: the combined results of several observers have shown numerous salient features of the variations in the spectrum of α Ceti; some of the same phenomena have been observed in χ Cygni; and changes in other stars have been indicated by shorter series of observations.

The present discussion will be confined largely to the bright lines, as observations of the continuous spectrum and of the absorption lines and bands do not yet afford a reliable basis for a description of the characteristic changes.² The data for the bright lines

¹ Contributions from the Mount Wilson Observatory, No. 200.

² The available observational material bearing on the absorption spectrum is not extensive, owing to the faintness of the stars, except at maximum, and to other circumstances. It is, however, definitely known that variations do take place, but

are not so complete as might be desired, but certain features are definitely established, and the general course of the changes is indicated. Much remains to be determined, however, especially in regard to the behavior of the spectrum at minimum brightness.

The new observational material described here refers to the stars in Table I. These are among the best known of the long-period variables, and all possess spectra strongly resembling that of α Ceti.

TABLE I
STARS OBSERVED

Star	α 1900	δ 1900	Magnitude
R Leonis.....	0 ^h 42 ^m 2	+11° 54'	4.6-10.5
R Hydrae.....	13 24.2	-22 46	4.0-9.8
R Serpentis.....	15 46.1	+15 26	5.6-13
X Ophiuchi.....	18 33.6	+8 44	6.5-9.0
χ Cygni.....	19 46.7	+32 40	4.0-13.5
T Cephei.....	21 8.2	+68 5	5.1-10.5

TABLE II
RESULTS OF HARVARD OBSERVATIONS

Star	Range	Period	$m-M$	Br.	Ft.
α Ceti.....	5.7 mags.	332 days	0.64	0.19	0.30
R Leonis.....	4.2	313	.55	.26	.28
R Hydrae.....	5.4	425	.57	.15	.32
R Serpentis.....	6.3	357	.61	.20	.28
X Ophiuchi.....	2.2	337	.45	.32	.44
χ Cygni.....	8.3	406	.54	.22	.17
T Cephei.....	3.9	387	0.46	0.23	0.27

Data from *Harvard Annals*, 57, 202, showing the character of the light-variation, are given in Table II, in which α Ceti is included for convenient comparison. The fourth column, " $m-M$," shows

apparent discrepancies or, perhaps, real differences between different stars, make it difficult to know which variations should be considered as characteristic. For instance it is stated by Adams and Joy, *Publications of the Astronomical Society of the Pacific*, 32, 163, 1920, and by Shane, *ibid.*, 32, 234, 1920, in confirmation of previous observations, that in α Ceti the absorption bands of titanium oxide increase in strength as the star wanes, but, in the case of X Ophiuchi, observations by the writer from May to September, 1920, show that the bands grew decidedly weaker while the brightness was decreasing. Although there are several possible explanations of this apparent discrepancy their discussion will not be taken up here.

the fraction of the period during which the light decreases; the fifth, "Br.," gives the fraction of the period during which the light is more than one-half as great as at maximum, and the last column, "Ft.," that fraction during which the light is less than twice as great as at minimum.

All the observations in Table III were made with one-prism spectrographs. The column "Phase" is the only one requiring explanation. This indicates for each plate the fraction of the period before (−), or after (+), light maximum. The photometric data were kindly supplied by the Harvard College Observatory.

R LEONIS

Jan. 16, 1920. This observation was made about 110 days after minimum and 60 days before maximum. The light-curve rose gradually for 70 or 80 days after minimum, and then much more rapidly.

May 30, 1920. This last observation was about 60 days before minimum.

R HYDRAE

A minimum occurred on April 27, 1920, magnitude 9.5. The first plate, on May 31, was taken 34 days after minimum, but the star had brightened only half a magnitude. Two bright lines which are the last to appear during the decreasing phase are still in evidence. The bright hydrogen lines, which in *Md* stars are weak or absent at minimum, have not yet appeared. The continuous spectrum is weak but is visible from $\lambda 4500$ toward longer wavelengths, being interrupted by wide absorption bands of titanium oxide. The second plate, taken on July 8, when the increase in magnitude since minimum had been 1.7 magnitudes, shows more continuous spectrum due to a generally more effective exposure, but the band absorption is very strong; $H\delta$ is a distinct emission line; $H\gamma$ is not seen, although continuous spectrum is visible in its neighborhood; $\lambda 4308$ is not seen; near $\lambda 4571$ the continuous spectrum is strong and the magnesium line of that wave-length, if present at all, is so blended with the background as to be indistinguishable.

TABLE III
OBSERVATIONS

Date	Mag.	Phase	Telescope	Camera	Exposure	Remarks
R Leonis						
1920 Jan. 16..	9.0	-0.19	100-inch	18-inch	164 ^m	Hubble observer
April 8..	6.4	+ .08	100	18	30	
11..	6.5	+ .09	60	18	140	
11..	6.5	+ .09	60	18	40	
May 5..	7.4	+ .16	60	18	152	
11..	7.6	+ .18	60	18	150	
30..	8.3	+0.24	100	18	80	
R Hydrae						
1920 May 31..	9.0	-0.33	100-inch	18-inch	214 ^m	
July 8..	7.8	-0.25	100	7	50	
R Serpentis						
1920 May 1..	7.6	+0.13	100-inch	18-inch	106 ^m	
30..	8.8	+ .20	100	18	184	
June 1..	8.8	+ .21	100	18	184	
July 8..	10.7	+0.32	100	7	160	
X Ophiuchi						
1919 June 9..	6.6	-0.03	60-inch	18-inch	24 ^m	Clouds
10..	6.6	- .02	60	18	30	
1920 April 8..	7.3	- .16	100	18	(70)	
May 1..	6.9	- .09	100	18	62	
4..	6.8	- .08	60	18	52	
31..	6.6	.00	100	18	80	
July 5..	7.0	+ .10	60	18	110	
6..	7.0	+ .11	100	18	90	
31..	7.5	+ .18	100	18	66	
Sept. 2..	8.3	+ .28	100	18	110	
27..	8.8	+ .35	100	18	180	Clouds
Oct. 27..	9.0	+0.44	100	18	90	
x Cygni						
1920 May 30..	8.2	-0.08	100-inch	18-inch	66 ^m	Benioff observer
June 1..	8.1	- .07	100	18	40	
30..	5.2	.00	60	18	125	
July 29..	5.8	+ .07	100	18	90	
Sept. 3..	6.5	+ .16	100	18	66	
26..	8.1	+ .21	100	18	180	
Oct. 26..	9.6	+0.29	100	18	180	

TABLE III—*Continued*

OBSERVATIONS

Date	Mag.	Phase	Telescope	Camera	Exposure	Remarks
T Cephei						
1917 Nov. 3..	7.6	-0.19	60-inch	18-inch	50 ^m	Joy observer
Dec. 3..	6.8	- .11	60	18	80	Joy observer
1918 Jan. 2..	5.8	- .04	60	18	60	Joy observer
Oct. 24..	7.2	- .27	60	18	150	Joy observer
1920 June 3..	8.0	+ .20	60	18	240	
July 5..	8.9	+ .28	60	18	364	
28..	9.4	+ .34	60	18	508	
Sept. 5..	10.1	+0.44	60	7	520	

R SERPENTIS

Maximum occurred on March 16, 1920, magnitude 6.1. The facts concerning the following minimum are not available at present. It appears, however, that the minimum brightness was not above the fourteenth magnitude. The spectroscopic observations described in Table IV cover about the central third of the decreasing branch of the light curve.

X OPHIUCHI

Maximum light and the decreasing phase are well covered by the observations. The last plate, taken on October 27, was probably about 20 days before minimum. The magnitude at this time was not more than a few tenths above the minimum magnitude. On April 8, 1920, H γ shows small contrast with the continuous spectrum; on May 1, this contrast is decidedly greater. On July 31, the titanium bands have a curious, blended appearance; the heads of the bands are not sharp, and as the absorption is less than on the preceding plates the bands are not so pronounced. The bands are still weaker in September. The decrease in intensity of H γ from September 2 to September 27 is striking; this line is practically absent from the spectrum on the latter date. The only emission line observed on the plate of October 27 is λ 4571, just at the red edge of which there appears a well-defined and rather strong *dark* line. This has developed within a month, as no

absorption was observed in this position on the preceding plate. A somewhat similar appearance of bright and dark portions of the same line has been observed by Stebbins¹ in α Ceti at λ 4308 and λ 4376, and by the writer in other stars in connection with the manganese line λ 4030. This phenomenon suggests that the well-known displacement of the dark lines with respect to the bright lines may be due in part to a reinforcement by a partially developed emission line of the continuous spectrum on the violet edge of the dark lines.

X Ophiuchi is a double star, Hu 198, with a measured separation of 0".2. The small light-range and some of the spectral peculiarities may thus be due to exceptional circumstances. The motions and light-variations of the components are being investigated by Van Biesbroeck at the Yerkes Observatory.²

RÉSUMÉ OF OBSERVED CHANGES IN THE BRIGHT LINES

Upon combining the data of Table IV with those previously available for Md stars, it is found that certain phenomena shown by the bright lines appear in substantially the same form in several stars. We thus conclude that these phenomena are characteristic of the variation of Md stars and that they are, therefore, of great importance in the study of the problems presented by these remarkable objects.

The changes exhibited by some of the more conspicuous bright lines are outlined in the following pages. In most cases several illustrative instances are cited to indicate the character and extent of the data upon which each statement rests. This collection of references and quotations may prove useful to anyone desiring to review the observational evidence.

Hydrogen lines.—The bright hydrogen lines are such conspicuous and characteristic features of Md spectra that they have been the means in many instances of the discovery of long-period variables at the Harvard Observatory. The variations of these lines have accordingly attracted more notice than any of the other spectral modifications presented by Md stars. Notes in the

¹ *Lick Observatory Bulletins*, 2, 93, 1903.

² December, 1920, meeting of the American Astronomical Society.

TABLE IV
INTENSITIES OF EMISSION LINES

Date	Phase	H γ	H δ	3005	H ϵ	H ζ	4202	4308	H γ	4373	4376	4571	H θ
R Leonis 313													
1920 Jan. 16...	-0.19	2
April 8...	+0.08	1	3	1	...	15	3	...	7	...	tr.
11...	+0.09	1-	3	0.5	...	15	4	...	7	...	0.5
May 5...	+0.16	1.5	3.5	1+	2	15	6	3	10	2	1.5	4	...
11...	+0.18	1.5	3.5	1	2	15	6	4	12	2	1	5	1
30...	+0.24	4	4	3+	6	2	1	5	tr
R Hydrae 425													
1920 May 31...	-0.33	1	3	...
July 8...	-0.25	2
R Serpentis 357													
1920 May 1...	+0.13	...	0.5	8	7	1
30...	+0.20	...	0.5	8	2+	2	7	1	0.5	2	2
June 1...	+0.21	...	0.5	tr	...	8	3	3	7	1	1	3	3
July 8...	+0.32	4+	3	4	4	0.5	1	4+	1.5
X Ophiuchi 387													
1919 June 9...	-0.03	6	4
10...	-0.02	7	4
1920 April 8...	-0.16	7	2
May 1...	-0.09	...	0.5	tr	...	9	4
4...	-0.08	...	0.5	8	4
31...	0.00	1-	2	1-	...	9	tr	...	6
July 5...	+0.10	6	1	...	4
6...	+0.11	0.5	1	0.5	...	7	2	...	6
31...	+0.18	2	1	...	4
Sept. 2...	+0.28	1	2-	1+	2	1-	...	2	...
27...	+0.35	tr	1	2	...
Oct. 27...	+0.44	1	...
x Cygni 406													
1920 May 30...	-0.08	...	2	7	2+	1
June 1...	0.07	...	1	6	2	0.5
30...	0.00	...	3	1-	...	15	tr	...	7	8
July 29...	+0.07	2	6	2	...	15	3	1	7	7
Sept. 3...	+0.16	1	3	1	...	15	3	3-	8	1	...	1	5
26...	+0.21	...	2	1	...	7	3	4	7	1	...	3	3
Oct. 26...	+0.29	tr	...	3	4	5	4	2	0.5	4	3
T Cephei 387													
1917 Nov. 3...	-0.19	3
Dec. 3...	-0.11	...	tr	15	4
1918 Jan. 2...	-0.04	2-	5	1	...	15	1	...	8	2
Oct. 24...	-0.27	4	1+
1920 June 3...	+0.20	2	2-	2+	4	0.5	...	4	0.5
July 5...	+0.28	2	4	2	0.5	...	6	0.5
28...	+0.34	1-	2	4	...
Sept. 5...	+0.44	0.5	1	2	...

Harvard publications show that in practically every Md star the relative intensities of $H\gamma$ and $H\delta$ are subject to wide variations.

Although the bright hydrogen lines (referring particularly to $H\gamma$ and $H\delta$) are usually of very great intensity, nevertheless it appears that near the phase of minimum light they are weak or absent from the spectrum.

To avoid repetition certain references will hereafter be indicated by number as follows:

- (1) Stebbins, "The Spectrum of α Ceti," *Lick Observatory Bulletins*, **2**, 78, 1903.
- (2) Eberhard, "On the Spectrum and Radial Velocity of χ Cygni," *Astro-physical Journal*, **18**, 198, 1903.
- (3) Merrill, "Spectroscopic Observations of Stars of Class Md," *Publications of the Astronomical Observatory, University of Michigan*, **2**, 45, 1916.
- (4) Adams and Joy, "Note on the Identification of Certain Bright Lines in the Spectrum of α Ceti," *Publications of the Astronomical Society of the Pacific*, **30**, 193, 1918.
- (5) Adams and Joy, "Changes in the Spectrum of Omicron Ceti," *Publications of the Astronomical Society of the Pacific*, **32**, 163, 1920.

ABSENCE OR WEAKNESS OF BRIGHT HYDROGEN LINES NEAR MINIMUM

- α Ceti. 1902, Oct. 4, to 1903, Jan. 5. Phase, $+0.36$ to $+0.64$. Reference (1). Hydrogen lines weaker than other bright lines, or not observed at all. "On 58A and 59A (1903, Jan. 2 and 5) the continuous spectrum was visible in the $H\gamma$ and $H\delta$ regions, but no trace of a bright line was seen in either case. The evidence furnished by these later plates therefore strengthens the conclusion that the bright lines disappeared at minimum."
- R Hydrae. 1920, May 31. Phase, -0.33 . Table IV. $H\gamma$ and $H\delta$ not observed although other bright lines and some continuous spectrum are present.
- X Ophiuchi. 1915, June 4. Phase, -0.38 . Reference (3). Faint continuous spectrum; no bright lines.
- 1920, Sept. 27 and Oct. 27. Phase, $+0.35$ and $+0.44$. Table IV. $H\gamma$ and $H\delta$ are very weak or absent.
- T Cephei. 1920, July 28 and Sept. 5. Phase, $+0.34$ and $+0.44$. Table IV. $H\gamma$ and $H\delta$ are not seen and are certainly much weaker than other bright lines.

Adams and Joy found that near the minimum of α Ceti the usual hydrogen lines were replaced by others having quite different characteristics. Whether this is typical behavior for long-period variables is not known.

HYDROGEN LINES AT MINIMUM

o Ceti. 1920, Jan. 15 to Feb. 6. Phase, $+0.51$ to $+0.57$. Reference (5).

"The hydrogen lines appear very diffuse and greatly displaced toward the red. The brightest of the three hydrogen lines on this photograph is $H\beta$, $H\gamma$ being next and $H\delta$ faint."

Shortly after minimum brightness the hydrogen lines $H\gamma$ and $H\delta$ appear faintly as emission lines, $H\gamma$ being at first much the weaker, so that for a time $H\delta$ may be seen alone. Both lines grow in intensity until maximum or after, but $H\gamma$ gains relatively to $H\delta$ so that during the decreasing phase, before both lines disappear, $H\gamma$ may become much the stronger.

In a brief summary of the general features of the spectra of long-period variable stars¹ as photographed with objective-prism spectrographs by the Harvard Observatory, Miss Cannon says: "Changes occur in the class of spectrum during the light variation, and in the relative intensities of the bright lines. In some cases, $H\delta$ appears first and becomes from ten to twenty times as bright as $H\gamma$. At maximum light, $H\gamma$ often reaches equality with $H\delta$. The relative intensities of these two lines often vary at different maxima of the same star." The results from slit spectrograms, as noted below, are in harmony with Miss Cannon's observations.

CHANGES IN RELATIVE INTENSITIES OF $H\gamma$ AND $H\delta$

o Ceti. 1915, Nov. 5, 12. Phase, -0.2 . Reference (3). "On Nov. 5, $H\gamma$ is so weak as not to be readily visible. It is easily seen on Nov. 12, but is not much stronger than the adjacent continuous spectrum. This is strikingly at variance with its appearance on my earlier plates and as observed by others."

1919, Autumn. Phase decreasing. Reference (5). "As the star grows fainter . . . the relative intensity of the hydrogen lines changes, $H\gamma$ becoming stronger with reference to $H\delta$."

R Leonis. 1920, April 8 to May 30. Phase, $+0.08$ to $+0.24$. Table IV. Relative intensities of $H\gamma$ and $H\delta$ changed from 7:15 to 6:4.

R Virginis. 1915, March 28 to May 7. Phase, -0.11 to $+0.16$. Reference (3). Relative intensities of $H\gamma$ and $H\delta$ changed from 3:4 to 2:1.

X Ophiuchi. 1920, April 8 to July 3. Phase, -0.16 to $+0.18$. Table IV. Relative intensities of $H\gamma$ and $H\delta$ changed from 2:7 to 4:2.

¹ *Popular Astronomy*, 27, 527, 1919.

χ Cygni. 1901, Aug. 2 to Nov. 23. Phase, -0.03 to $+0.24$. Reference (2).

"From Aug. 2 until Sept. 19 $H\delta$ was considerably brighter than $H\gamma$; from Oct. 3 to 15 $H\gamma$ and $H\delta$ differed little from each other; on Oct. 26 they were equal; and on Nov. 9 and 23 $H\gamma$ was brighter than $H\delta$."

1920, May 30 to Oct. 26. Phase, -0.08 to $+0.29$. Table IV. Relative intensities of $H\gamma$ and $H\delta$ changed from $2+ : 7$ to $4 : 3$.

The foregoing are instances of considerable variation in the hydrogen lines occurring in a single light cycle. Many more examples can be found by comparing observations in different cycles, but deductions from such comparisons must be made with circumspection since it is not safe to assume that the behavior of the spectrum of a long-period variable is identically repeated in different cycles.

The relative intensities of the hydrogen lines near maximum are very different from laboratory values. The most striking discrepancies are the great strength of $H\delta$ and of numerous ultra-violet lines, and the relative weakness of $H\beta$ and $H\epsilon$. Miss Maury¹ and C. D. Shane² have called attention to the relative weakness of $H\kappa$, $H\lambda$, and $H\mu$ in α Ceti; the same phenomenon has been observed partially in other Md spectra through the fact that the hydrogen series appears to stop abruptly with $H\epsilon$. Other peculiarities of the hydrogen series are described by Shane. By combining the results of several observers it appears that a bright $H\alpha$ line may or may not be present in the spectrum of α Ceti. Unfortunately this line has not been observed systematically over any considerable period of time. The variation in $H\beta$ is thought by Father Cortie³ to be connected with the brightness of the maximum. According to him, at a bright maximum the titanium oxide bands are especially weak and $H\beta$ especially strong. This same correlation between $H\beta$ and the titanium bands appears also in comparing the average characteristics of different stars.⁴ $H\epsilon$ (though seldom observed) may increase relatively to $H\delta$ and $H\gamma$ during the decline in brightness.

¹ *Harvard Annals*, **28**, 45, 1897.

² *Publications of the Astronomical Society of the Pacific*, **32**, 234, 1920.

³ *Scientific American Supplement*, **85**, 21, 1918.

⁴ *Publications of the Astronomical Observatory, University of Michigan*, **2**, 63, 1916.

CHANGES IN RELATIVE INTENSITY OF He

α Ceti. 1902, June 27 to Sept. 22. Phase, $+0.07$ to $+0.33$. Reference (1).

Relative intensities of H δ , He, and H ζ changed from 420:4:30 to 40:10:7.

R Leonis. 1920, April 8 to May 11. Phase, $+0.08$ to $+0.18$. Table IV.

Relative intensities of H δ , He, and H ζ changed from 15:-:3 to 15:2:3.5.

Observations by Shane¹ at the Lick Observatory have revealed the interesting fact that in variables of Class N the characteristic phenomena connected with the bright hydrogen lines are much the same as in variables of Class Md. It appears, however, that in Class N the bright hydrogen lines reach their greatest intensities earlier in the light cycle, and are less persistent after maximum light than in the Md stars.

Bright lines not due to hydrogen.—Besides the hydrogen series there are other bright lines subject to very great changes in intensity during the light cycle. The available data, though rather fragmentary, make clear a certain system in these changes which is undoubtedly followed by several, and probably by many or all, Md long-period variables. Accordingly an attempt will be made in the following paragraphs to indicate some of the phenomena connected with these various bright lines as typical of Md variation.

λ 3905.—This line, thought to be due to silicon (3905.51 I.A.), is (excepting hydrogen) one of the strongest and most persistent of the emission lines during the time that the star is bright. After maximum it seems to gain relatively to the hydrogen lines, but it is possible that this increase arises from the diminution of the latter. Its behavior near the minimum phase is not known for it has usually been photographed only when the continuous spectrum and the bright hydrogen lines are strong. It appears certain, however, that λ 3905 does not grow stronger toward minimum to any such degree as do λ 4308 and λ 4571 (see below). Further observations of this line are much to be desired.

λ 4202.—This iron line (4202.033 I.A.) has very frequently been observed as an emission line in Md spectra. Appearing about the time of maximum, it grows steadily stronger during the first part of the decline until it may equal or surpass H γ and H δ . This increase does not continue throughout the decreasing light phase,

¹ *The Spectra of Certain Class N Stars, Lick Observatory Bulletins*, 10, 79, 1920.

as toward minimum this line is in turn exceeded in intensity by $\lambda 4308$ and $\lambda 4571$.

CHANGES IN RELATIVE INTENSITY OF $\lambda 4202$

- α Ceti. 1902, June 27 to Oct. 6. Phase, $+0.07$ to $+0.37$. Reference (1).
Relative intensities of $H\gamma$, $\lambda 4202$, and $H\delta$ varied from 420:10:420 to 13:40:13.
- R Leonis. 1920, Jan. 16 to May 30. Phase, -0.19 to $+0.24$. Table IV.
Relative intensities of $H\gamma$, $\lambda 4202$, and $H\delta$ varied from $-:-2$ to 6:4:4.
- χ Cygni. 1920, June 30 to Oct. 26. Phase, 0.00 to $+0.29$. Table IV.
Relative intensities of $H\gamma$, $\lambda 4202$, and $H\delta$ varied from 7:-:15 to 4:4:3.
- T Cephei. 1920, June 3 to July 28. Phase, $+0.20$ to $+0.34$. Table IV.
Relative intensities of $H\gamma$, $\lambda 4202$, and $H\delta$ varied from 4:2:-2 through 2:2:- to -:1:-.

$\lambda 4308$.—This line, apparently belonging to the iron spectrum (4307.909 I.A.), becomes bright at a later phase than the somewhat similar line $\lambda 4202$, and grows stronger than $\lambda 4202$ if the observations are continued a sufficient time past maximum. Near maximum $\lambda 4308$ is sometimes observed as an absorption line, usually of low intensity.

CHANGES IN RELATIVE INTENSITIES OF $\lambda 4202$ AND $\lambda 4308$

- α Ceti. 1902, June 27 to Oct. 26. Phase, $+0.07$ to $+0.43$. Reference (1).
Relative intensities of $\lambda 4202$, $\lambda 4308$, and $H\gamma$ varied from 10:-:420 through 19:12:190 to 40:60:13. $\lambda 4308$ appeared between July 22 and 29; phase, about $+0.16$.
- R Leonis. 1920, April 8 to May 30. Phase, $+0.08$ to $+0.24$. Table IV.
Relative intensities of $\lambda 4202$, $\lambda 4308$, and $H\gamma$ varied from 3:-:7 to 4:3+:6.
- R Serpentinis. 1920, May 30 to July 8. Phase, $+0.20$ to $+0.32$. Table IV.
Relative intensities of $\lambda 4202$, $\lambda 4308$, and $H\gamma$ varied from 2+:2:7 to 3:4:4.
- χ Cygni. 1901, Aug. 24 to Nov. 23. Phase, $+0.02$ to $+0.17$. Reference (2). " $\lambda 4308$ increased in brightness the fainter the star became."
1920, July 29 to Oct. 26. Phase, $+0.07$ to $+0.29$. Table IV. $\lambda 4308$ appeared as an emission line between June 30 and July 29. Relative intensities of $\lambda 4202$, $\lambda 4308$, and $H\gamma$ varied from 3:1:7 to 4:5:4.
- T Cephei. 1920, June 3 to July 28. Phase, $+0.20$ to $+0.34$. Table IV.
Relative intensities of $\lambda 4202$, $\lambda 4308$, and $H\gamma$ varied from 2-:2+:4 to 1-:2:-. Compare with plate of 1918, Jan. 2, on which the intensities were 8:-:1. Compare also with $H\delta$. Intensities of $\lambda 4308$ and $H\delta$ on 1917, Dec. 3, 1918, Jan. 2, were -:15 and on 1920, July 5, 4:-. The relative intensities of these two lines must have changed 250 or more times.

$\lambda 4571$.—This line, which is probably a low-temperature magnesium line (4571.114 I.A.), does not appear until a later epoch than $\lambda 4202$ and $\lambda 4308$, but gains in relative strength as minimum is approached. In several instances it has finally become the strongest line in the photographic spectrum. This is probably typical behavior. This line is usually observed near the maximum phase as a weak absorption line.

CHANGES IN RELATIVE INTENSITIES OF $\lambda 4308$, $H\gamma$, AND $\lambda 4571$

- α Ceti. 1902, July 22 to Oct. 26. Phase, $+0.14$ to $+0.43$. Reference (1).
Relative intensities of $\lambda 4308$, $H\gamma$, and $\lambda 4571$ changed from 12:190:- to 60:13:75.
- 1918, March 2. Phase, $+0.44$. Reference (4). "This line ($\lambda 4571$) is the most intense of any on the photograph with the exception of those due to hydrogen." This precise condition was not shown by any of Stebbins' photographs although it was approached on Aug. 11, Aug. 25, and Sept. 6, 1902.
- R Leonis. 1920, May 5 to May 30. Phase, $+0.16$ to $+0.24$. Table IV.
Relative intensities of $\lambda 4308$, $H\gamma$, and $\lambda 4571$ changed from 3:10:4 to 3+:6:5.
- R Hydrae. 1920, May 31. Phase, -0.33 . Table IV. Only two bright lines are visible, $\lambda 4308$ and $\lambda 4571$. Compare with the observation of July 8, and with the Ann Arbor observations.
- R Serpentis. 1920, May 30 to July 8. Phase, $+0.20$ to $+0.32$. Table IV.
Relative intensities of $\lambda 4308$, $H\gamma$, and $\lambda 4571$ changed from 2:7:2 to 4:4:4+.
- χ Cygni. 1920, July 29 to Oct. 26. Phase, $+0.07$ to $+0.29$. Table IV.
Relative intensities of $\lambda 4308$, $H\gamma$, and $\lambda 4571$ varied from 1:7:- to 5:4:4.
- T Cephei. 1920, June 3 to Sept. 5. Phase, $+0.20$ to $+0.44$. Table IV.
Relative intensities of $\lambda 4308$, $H\gamma$, and $\lambda 4571$ changed from 2+:4:4 to 1:-:2.

GENERAL DISCUSSION

It should not be inferred that the foregoing references represent all the known observational material bearing on each line. They have been selected as the most complete and reliable illustrations of the changes under discussion. Fragmentary observations indicate that the same course of spectral variation is followed in other stars, and in these same stars in other light cycles.

In Figure 1 an attempt has been made to display graphically the changes in the intensities of the lines $H\gamma$, $H\delta$, and $\lambda 4202$, $\lambda 4308$, and $\lambda 4571$, as related to the light phase. The relationships

depicted are believed to be typical of long-period variables of Class Md. It is clear that considerable differences exist between the spectra of different stars at any given phase, but all stars of the α Ceti type which have been adequately observed have shown, with no very great variations, the changes in spectrum indicated in Figure 1. The phases at which certain lines appear or disappear may vary somewhat in different stars. Figure 1 is intended to represent the average behavior of the small group of stars observed, and may require modification when more complete data become available.

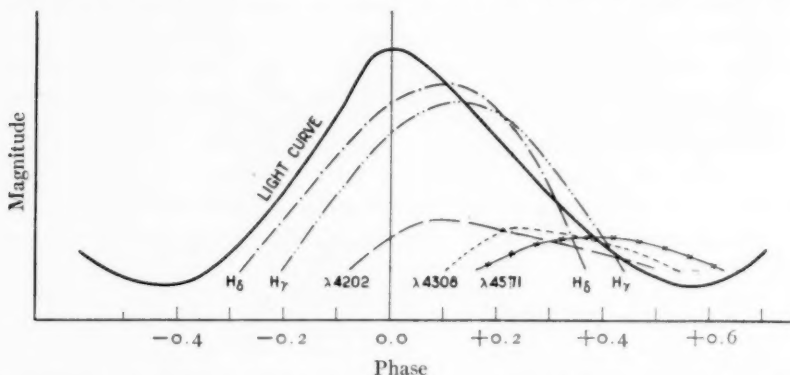


FIG. 1.—Intensities of bright lines in Md spectra in relation to the variation in total light. The light-curve, which is the mean curve for α Ceti, R Leonis, R Hydrae, χ Cygni, and T Cephei, has an amplitude of 5.3 magnitudes. The amplitudes of the other curves are not determined.

The method of comparing observations of different stars by referring them to the light phases counted from maximum to maximum is obviously but a first approximation. For an exact comparison of the behavior of different stars the light-curves cannot be assumed to be of the same shape, or to be identically repeated in different cycles. In a final discussion the circumstances of any particular cycle should be recognized either by counting the phase from maximum to *minimum* instead of from maximum to maximum, or by dropping the time relation and using instead the *magnitude* when observed, as compared with the magnitudes at the maximum and minimum of the same cycle.

In addition to the lines which have been discussed in the preceding pages a large number of other less conspicuous bright lines

have been observed in Md spectra, including $\lambda\lambda$ 3852, 3967, 4007, 4030, 4102, 4138, 4178, 4215, 4233, 4373, 4376, and 4512, all of which appear subject to variation. The intensities of λ 4373 and λ 4376 are found in Table IV. After some further observations have been secured several of these lines can probably be grouped according to their behavior and compared with the stronger lines λ 4202, λ 4308, and λ 4571. They may thus prove useful aids in interpretation. It is hoped to discuss them in a future contribution.

Adams and Joy have pointed out that "the bright lines appearing in the spectrum of α Ceti near minimum of light are lines which are strengthened at low temperatures."¹ This is undoubtedly correct, but it appears that some other cause in addition to low temperature must be operative to produce the observed peculiarities. The magnesium line 4571 is indeed an extreme low-temperature line and its behavior strongly suggests low temperature as the prevailing condition near minimum. With λ 4202 and λ 4308 and some other lines, however, the case is not so clear. There is no apparent reason why certain of these lines should be so strongly accentuated as compared with other lines of the same elements in the same temperature class² and having comparable intensities. It will probably be well to reserve discussion of temperature and other physical conditions until the fainter emission lines mentioned in the foregoing paragraph have been more extensively observed.

It is believed that the spectral changes described in this paper have such intimate connection with the light variations of Md stars that a knowledge of them is quite essential to a correct general interpretation of the phenomena exhibited by stars of this class. More complete observations of the bright lines, of the dark lines and bands, and especially of the spectrum near minimum will be required as a basis for a satisfactory discussion of the conditions prevailing in these stars and of the causes of variation. The writer is inclined to favor the "veil theory" of variation as outlined a few years ago,³ but even if this be correct much amplification will be needed.

¹ *Publications of the Astronomical Society of the Pacific*, **30**, 193, 1918.

² King, *Mt. Wilson Contr.*, No. 66; *Astrophysical Journal*, **37**, 309, 1913.

³ *Publications of the Astronomical Observatory, University of Michigan*, **2**, 70, 1916.

o Ceti, the brightest of the long-period variables and having a period which differs appreciably from a year, offers the strategic point for an observational attack on the physical problems of Md variation, provided we are justified in assuming that the remarkable spectral features of this star and their variations are not isolated peculiarities but may be accepted as typical Md phenomena. One consequence of the present discussion may be increased confidence that this is the case.

MOUNT WILSON OBSERVATORY
December 1920

THE ORBITS OF SEVEN SPECTROSCOPIC BINARIES¹

By R. F. SANFORD

ABSTRACT

Seven spectroscopic binaries, Boss 373, OΣ 82, Lalande 29330, Companion to α Herculis, 205 Draconis, Boss 5591, Lalande 46867.—The elements of the orbits have been derived from 15 to 30 spectrograms of each, most of them made with an 18-inch camera and the 60-inch or 100-inch reflector. In the case of the first, fifth, and sixth binaries, both components were measured. The periods are each about 4 days except for the fourth and seventh, which are 51.6 and 6.72 days respectively. Radial-velocity diagrams are shown in Figures 1-7. The spectroscopic parallax, proper motion, and absolute magnitude of each, as furnished by Adams and Joy, are given in Table I. Lalande 29330 and 46867 are particularly interesting because of low luminosity, +6, and late spectral class, K. Lalande 46867 is very similar in spectroscopic peculiarities to σ Geminorum.

α Herculis and companion.—The radial velocity of the center of mass of the companion is -37.2 km/sec., whereas that of α Herculis is -32.2 km/sec.; and from the parallax they are separated at least 250 astronomical units. These facts indicate that they probably form merely an optical pair.

Probable eclipsing variable star.—The data for 205 Draconis suggest that it would be a favorable case to examine for variability caused by eclipse. Its magnitude is 7.2 and spectral class F2.

This paper deals with the derivation of the elements of the orbits of the seven spectroscopic binary stars listed in Table I.

TABLE I

Name	Mag.	α 1900	δ 1900	Spectral Class	Abs. Mag.	μ	π _{sp.}	No. Rev.
Boss 373 = Σ 145..	6.3	1 ^h 36 ^m	+25° 14'	F3-F3	+2.6	0.135	0.018	161
OΣ 82	7.0	4 17	+14 49	F9	+4.1	0.096	0.024	194
Lalande 29330...	8.5	16 1	+10 57	K0	+6.0	0.491	0.032	350
Comp. α Herculis-								
Boss 4374.....	5.4	17 10	+14 30	F9	+1.7	0.032	0.018	7
205 Drac. = β 971..	7.2	18 45	+49 19	F2-F2	+1.4	0.005	119
Boss 5591 = π. 267	6.9	21 40	+28 19	A9-A9	+2.3	0.061	0.009	103
Lalande 46867...	7.3	23 50	+28 5	K2	+6.0	0.573	0.055	179

The last four columns of the table give, respectively, the absolute magnitude, proper motion, spectroscopic parallax, and number of revolutions of the binary in its orbit between the first and last observations. Six of the stars were announced as binaries in a note by Adams and Joy,² who have also furnished from manuscript the data for absolute magnitude, proper motion, and spectroscopic parallax.

¹ Contributions from the Mount Wilson Observatory, No. 201.

² Publications of the Astronomical Society of the Pacific, 31, 41, 1919.

The spectroscopic parallaxes for the first, fifth, and sixth binaries may be considerably in error, since blends of the spectra of the two stars may affect the relative intensities of the lines used for the determination of the absolute magnitudes. The spectral class in these three cases is indicated for both components. The elements of three of the above binaries have been previously announced by the writer.¹

The spectrograms upon which the orbits depend were obtained with spectrographs of one-prism dispersion and an 18-inch camera, the only exceptions being a few plates of Lalande 29330, noted in Table IV, which were obtained with a 7-inch camera. Both the 60-inch and the 100-inch reflector were used: in all cases the letter prefixed to the plate number indicates the telescope—a Greek γ for the 60-inch series, and the letter C for the 100-inch series. Except where noted in the remarks, Seed 30 plates were used.

An effort was made in each case to determine the period as soon as possible and thereafter to secure with the minimum number of plates a reasonably good distribution over the velocity-curve. In some cases this has led to the determination of the elements from rather fewer plates than has usually been the custom. But no great increase in accuracy would have been possible without the addition of a very considerable number of observations and it was considered better to spend the time in the investigation of other orbits rather than accumulate observations on a few binaries.

The tables of observations accompanying the individual discussions are similar, each giving the plate number, date, Greenwich mean time of mid-exposure, the phase from periastron, the measured velocity, the residuals from the final elements, the weights assigned when any distinction has been made among the plates of a binary, and finally such remarks as seemed necessary. When the spectra of both components have been measured on the same plate two columns for velocities and two sets of residuals are given in the tables.

A few general remarks will make clear the procedure which has been followed and will avoid unnecessary repetition.

Since, in every case but one, at least 100 revolutions of the stars in their orbits separate the first and last observations, it seemed

¹ *Publications of the Astronomical Society of the Pacific*, 32, 192, 1920.

justifiable to assume that the period already determined needed no correction. Except for Lalande 29330 and the companion to α Herculis, the preliminary period has, therefore, been taken as final. The freehand velocity-curves drawn through the plotted observations were used to determine preliminary elements by Russell's method.¹ When a velocity diagram included the curves of both components, the preliminary elements used were based upon both curves, excepting of course the semi-amplitude of velocity variation, K . For such binaries the elements were corrected by the method of least squares, using the equations of condition given by Harper,² in which a single set of normal equations is derived from the observed velocities of both primary and secondary. Assuming the period to be known and noting that the angles of periastron for primary and secondary differ by 180° , it is readily seen that there are six independent elements to be corrected, so that the normal equations involve six unknown quantities. For the binaries showing only one spectrum, the equations adapted by Schlesinger³ were used to derive corrections to the preliminary elements.

In all cases where the elements are given for two components, those which are common to the two have not been repeated. Those which belong to the secondary alone are distinguishable by the subscript 1.

A radial velocity diagram has been drawn for each binary (Figs. 1-7). The zero for the abscissae corresponds to the epoch of periastron. The curves are based upon the elements adopted as final; the individual velocities are represented by circles with radii equal to the probable error of a velocity of unit weight; and a broken line indicates the velocity of the center of mass (γ).

The details for the individual stars are briefly as follows:

BOSS 373

This star is the brighter component of the double Σ 145. The fainter component is of the eleventh magnitude, $11''$ distant, in

¹ *Astrophysical Journal*, **40**, 282, 1914.

² *Publications of the Dominion Observatory*, **1**, 327, 1914.

³ *Publications of the Allegheny Observatory*, **1**, 33, 1908.

p.a. 32° . Measures separated by 35 years show no evidence of orbital motion.

Table II gives the nineteen spectrograms available for the derivation of the elements, and three others subsequently obtained. Fifteen of these gave velocities for both components and showed

TABLE II
OBSERVATIONS OF BOSS 373

PLATE NO.	DATE	G.M.T.	PHASE	VELOCITY		O-C		Wt.
				Prim.	Second.	Prim.	Second.	
				km/sec.		km/sec.		
γ 7272...	1918 Aug. 27	23 ^h 57 ^m	3 ^d 88 ^o	-25.2	+44.9	+6.1	+1.4	0.5
7379...	Sept. 20	22 53	1.228	+68.9	-60.5	+5.1	-0.7	1.0
7412...	23	22 37	4.217	+12.8		+0.5	+17.8	0.5
7448...	Oct. 17	21 26	1.560	+39.8	-28.4	+5.5	-0.6	0.5
7582...	Nov. 21	18 43	0.969	+76.1	-74.0	-5.0	+4.7	1.0
7596...	22	17 56	1.934	+10.2		+12.9	-0.7	0.5
7603...	Dec. 11	17 22	3.181	-76.3	+90.6	-3.1	+1.5	1.0
7618...	12	18 19	4.212	+10.4		-1.2	+14.7	0.5
7628...	13	18 16	0.775	+79.5	-83.4	-9.1	+3.4	1.0
7639...	14	16 30	1.702	+10.2		-9.5	+23.2	0.5
7646...	15	19 22	2.820	-68.4	+78.4	-2.1	-3.1	1.0
7662...	18	16 34	1.269	+61.2	-67.0	+0.8	-10.8	1.0
7739...	1919 Jan. 11	16 48	3.106	-74.5	+87.2	-1.4	-1.8	1.0
7769...	15	18 05	2.724	-58.6	+78.1	+3.3	+1.4	1.0
7782...	17	16 20	0.217	+72.1	-68.5	+3.8	-3.8	1.0
7850...	Feb. 8	15 45	0.018	+47.3	-47.6	+1.6	-7.4	1.0
8942...	1920 Jan. 6	15 50	3.851	-35.1	+47.5	-0.6	+0.4	0.5
8958...	9	16 07	2.429	-50.7	+55.6	-7.1	-1.3	0.5
8991...	Feb. 6	15 02	3.775	-44.8	+53.5	-2.1	-2.4	1.0
9428...	Aug. 3	23 48	1.315	+60.8	-44.2	+5.3	+7.7
9502...	29	21 45	0.622	+81.7	-60.2	-6.9	+27.7
9519...	Sept. 1	22 24	3.657	-55.4	+64.3	-1.0	-3.2

that both are of spectral class F₃. On the other four plates the velocities of the two components differ so little that with the dispersion employed the lines could not be separately measured. Mr. Joy had obtained 4.4 days as an approximate period, which soon led to 4.43474 days as the best value for the assembly of the observations about a single epoch.

The following preliminary elements were found:

$$\begin{aligned}
 P &= 4.43474 \text{ days} & K &= 79 \text{ km/sec.} \\
 e &= 0.12 & K_1 &= 89 \text{ km/sec.} \\
 \omega &= 270^\circ & T &= \text{J.D. } 2,421,940.65 \\
 \omega_1 &= 90^\circ & \gamma &= +5 \text{ km/sec.}
 \end{aligned}$$

The four plates on which the lines of the two components are blended were combined into two normal places; otherwise each plate furnished a conditional equation for each component, thus giving a total of thirty-four equations. The weights depend upon the separation of the lines of the two components and refer equally to the velocities for primary and secondary. Two least-squares solutions were made with the corrections to six elements as unknowns. In the second solution the corrections were small for all except ω and T . A third solution was then made assuming all elements to be definitive except these two. This solution gave

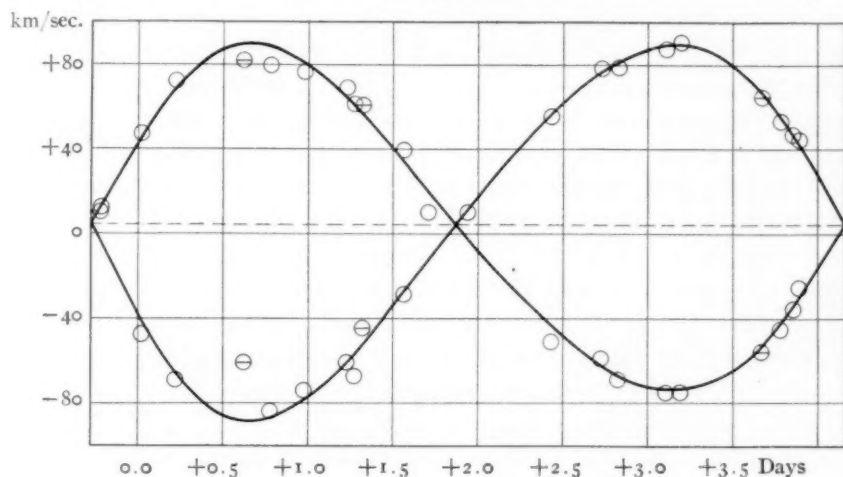


FIG. 1—Velocity-curves of Boss 373

only slight corrections. The second solution (six unknowns) and the third (two unknowns) yield elements from which nearly identical residuals result. In either case the quantity Σpv^2 is little more than 50 per cent of its value for the preliminary elements. Applying the corrections obtained by the three solutions and deriving the probable errors from the second solution we have the following final elements:

$P = 4.43474$ days	$T = \text{J.D. } 2,421,940.9865 \pm 0^d 0053$
$e = 0.108 \pm 0.011$	$\gamma = +4.6$ km/sec.
$\omega = 295^\circ 58 \pm 5^\circ 02$	$a \sin i = 4,942,100$ km
$\omega_1 = 115^\circ 58 \pm 5^\circ 02$	$a_1 \sin i = 5,371,800$ km
$K = 81.5 \pm 1.3$ km/sec.	$m \sin^3 i = 1.16 \odot$
$K_1 = 88.6 \pm 1.3$ km/sec.	$m_1 \sin^3 i = 1.06 \odot$

The probable error of a radial velocity of unit weight based upon all the residuals, including those from the four plates with blended spectra, is ± 3.5 km/sec. The three observations obtained after the elements were derived have been plotted upon the velocity diagram (Fig. 1) as barred circles. The velocity for the secondary star from $\gamma 9502$ is discordant. This plate is strongly exposed. The velocity derived from $H\gamma$ alone is -81 km/sec., which agrees fairly well with the curve.

O Σ 82

The star is a visual binary in comparatively rapid motion, the position angle having changed about 180° since discovery. The elements by different computers still differ widely, however, the periods ranging from 90 to 158 years. The last available observations of the visual pair were kindly furnished by Dr. Aitken. In 1916 the position angle was about 75° and the separation about $0''.7$. During the period covered by the spectrograms it was not possible to separate the brighter star from the fainter on the slit, even with the 100-inch reflector. Consequently light from both visual components entered the slit, but in the time needed to get a good spectrogram, component *B*, two magnitudes fainter than *A*, could produce no appreciable effect upon the photographic plate. Consequently the spectroscopic binary here studied is star *A*.

The data of the fifteen plates available are given in Table III. The five plates taken on successive nights showed the period to be close to four days, and a subsequent adjustment of all fifteen observations was best made by using $P = 4.00000$ days.

Since the period is an exact number of days, it was impossible to observe all parts of the velocity-curve. Fortunately two of the regions that could be observed correspond to maximum and minimum radial velocity. Little would have been gained by multiplying observations of the four parts of the curve that can be represented, and it was therefore decided to derive the best elements possible from the fifteen plates available.

The curve resulting from an assembly of the observations about a single epoch gave the following provisional elements:

$P = 4.00000$ days	$K = 35.0$ km/sec.
$e = 0.15$	$T = \text{J.D. } 2,422,274.680$
$\omega = 0^\circ 00$	$\gamma = +37.4$ km/sec.

These were once corrected by the method of least squares applied to the equations of Schlesinger. The resulting elements and their probable errors are adopted as final for the data available:

$$\begin{aligned}
 P &= 4.00000 \text{ days} & K &= 36.1 \pm 0.7 \text{ km/sec.} \\
 e &= 0.060 \pm 0.020 & T &= \text{J.D. } 2,422,274.8123 \pm 0^d2400 \\
 \omega &= +12^\circ 74' \pm 13^\circ 56' & \gamma &= +37.4 \text{ km/sec.} \\
 a \sin i &= 1,980,000 \text{ km} \\
 \frac{m_1^3 \sin^3 i}{(m+m_1)^2} &= 0.0193 \odot
 \end{aligned}$$

TABLE III
OBSERVATIONS OF Ω 82

Plate No.	Date	G.M.T.	Phase	Velocity	O-C
				km/sec.	km/sec.
γ 6511.....	1917 Dec. 30	18 ^h 43 ^m	2 ^d 968	+45.0	+3.6
6655.....	1918 Jan. 30	15 44	1.844	+4.3	+0.9
6761.....	Mar. 22	15 45	0.844	+34.6	-1.5
7451.....	Oct. 18	0 08	2.194	+7.4	-0.3
8782.....	1919 Oct. 13	0 25	2.205	+7.4	-0.6
8850.....	Nov. 10	20 38	3.048	+43.2	-2.7
8858.....	11	22 13	0.114	+74.8	+2.4
8865.....	12	17 25	0.914	+31.5	-0.6
C 213.....	13	22 18	2.117	+1.0	-5.1
C 217.....	14	19 26	2.998	+42.5	-0.6
γ 8908.....	Dec. 9	20 57	0.061	+73.8	+0.2
γ 8922.....	13	16 49	3.889	+76.2	+0.7
C 244.....	1920 Jan. 12	19 07	1.985	+9.6	+5.4
C 245.....	13	15 30	2.833	+34.4	+0.4
C 282*.....	Feb. 11	15 06	3.817	+74.5	-0.9
γ 3804†.....	1914 Oct. 26	22 10	2.110	+11.3	+5.4

* Seed 23.

† Not used for orbit.

The quantity Σpv^2 corresponding to these elements is less than 50 per cent of its value for the preliminary elements. The probable error of a velocity of unit weight is ± 2.0 km/sec. The size of the probable error, the sum of the residuals, the small number of observations, and the uncertainties that enter because of the limited portions of the velocity-curve which can be observed, render further approximations inadvisable.

After the elements had been derived it was discovered that a spectrogram, listed at the end of Table III, had been taken on October 26, 1914. The velocity from this plate has been plotted

in the diagram (Fig. 2) as a barred circle. Four hundred and sixty-one revolutions of the binary separate this plate from the epoch of the velocity diagram. If the period were lengthened by 0.00050 of a day, this velocity would fall upon the curve and the distribution of the other velocities would hardly be affected; but as it stands the representation is as good as for some of the other plates, and the period has been left as previously found. After the lapse of several years it will be possible to make any slight change in the period that may be necessary.

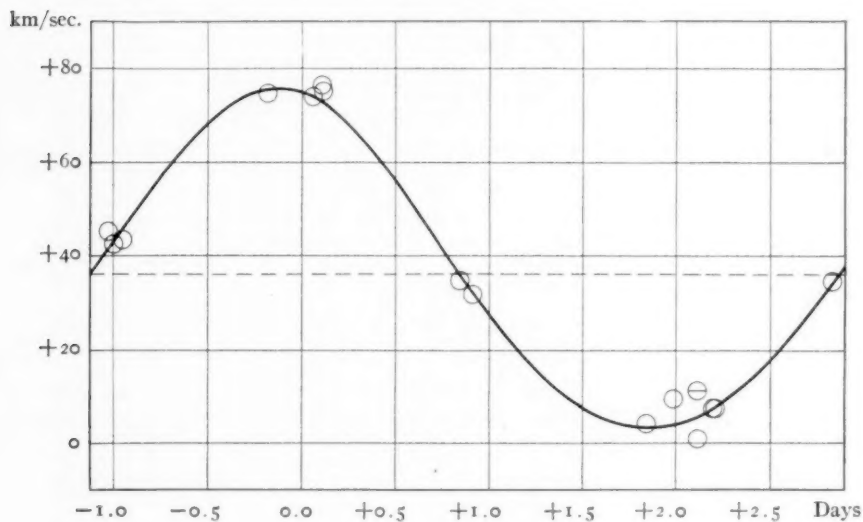


FIG. 2.—Velocity-curve of OΣ 82

From spectrograms of 19 members of the Taurus stream, Adams, Joy, and Strömberg¹ find +39 km/sec. as the mean radial velocity of the group. The velocity of the center of mass of OΣ 82 *A* found above for 1919 is +37.4 km/sec. Bearing in mind that the period of the visual binary is certainly more than 50 years and that the mass of the visual component *B* almost certainly is less than that of *A* (it is two magnitudes fainter), the radial velocity of component *A* (+37.4 km/sec.), the proper motion, and the position seem to designate the system as a member

¹ *Publications of the Astronomical Society of the Pacific*, 32, 195, 1920.

of the Taurus stream. The mean value of the parallax for the group from the determinations of Boss and of Kapteyn, and from the spectroscopic results, giving equal weight to each, is $+0''.023$. If we call M_A the mass of visual component A , and M_B the mass of visual component B , we can compute the value of $M_A + M_B$ for each of the three orbits given by Hussey.¹ Further, if we assume the velocity of the center of mass of the visual binary to be $+39$ km/sec., the velocity of B can be derived from the velocity of A ($+37.4$ km/sec.) and the parallax, $+0''.023$. From the velocities of A , B , and the center of mass the ratio $\frac{M_B}{M_A}$ can be calculated for each set of elements. The values $(M_A + M_B)$ and $\frac{M_B}{M_A}$ then give M_A and M_B corresponding to each orbit. The quantity $\frac{m_1^3 \sin^3 i}{(m + m_1)^2}$, derived from the spectroscopic observations, contains $m + m_1 = M_A$; hence, if we substitute successively the values of i given for each of the three visual orbits, M_A may be split up into the two parts m and m_1 . The results are as follows:

Unit is mass of sun	Orbit by	Glazenapp	Gore	Hussey
	$M_A \begin{cases} m \\ m_1 \end{cases}$	$1.94 \begin{cases} 1.17 \\ 0.77 \end{cases}$	$4.45 \begin{cases} 3.44 \\ 1.01 \end{cases}$	$6.19 \begin{cases} 5.04 \\ 1.15 \end{cases}$
	M_B	0.78	0.68	0.93

These values are given with the clear understanding that they are based upon the assumption that the radial velocity of the center of mass of OΣ 82 differs in nowise from the mean value for 19 other stars of the group. Furthermore no one of the three sets of visual elements is probably correct, nor does it follow that i has even approximately the same value in the spectroscopic orbit as in the visual orbit.

Note added February 17, 1921.—Exceptionally good conditions of seeing upon the night of November 23, 1920, made it possible to separate stars A and B of the visual binary OΣ 82 with the 100-inch telescope, and an attempt was made to secure the spectrum of star B separated from that of star A . As the position angle, 75° , is not favorable, guiding was difficult. The velocity derived from

¹ *Publications of the Lick Observatory*, 5, 60, 1901.

measurement of this plate is $+38.6$ km/sec. The velocity predicted from the elements of the spectroscopic binary, star *A*, is $+5.0$ km/sec. Hence there is no question but that the spectrum obtained is that of *B*. Furthermore this velocity is that of the Taurus stream which was used in the discussion of the probable masses. The spectral class of *B* is not greatly different from that of *A*, though probably slightly later.

LALANDE 29330

This binary is of particular interest because of its low luminosity (abs. mag. = $+6.0$) and late spectral class, Ko. The twenty plates upon which its orbit is based are summarized in Table IV. Spectrograms upon five succeeding nights led to a trial of periods around four days. A smooth velocity-curve was obtained when $P=4.28495$ days was used. This yielded the preliminary elements:

$$\begin{aligned} P &= 4.28495 \text{ days} & K &= 38.2 \text{ km/sec.} \\ e &= 0.10 & T &= \text{J.D. } 2,422,418.619 \\ \omega &= 134^\circ & \gamma &= -60.3 \text{ km/sec.} \end{aligned}$$

TABLE IV
OBSERVATIONS OF LALANDE 29330

Plate No.	Date	G.M.T.	Phase	Velocity	O—C	Wt.
				km/sec.	km/sec.	
74886*	1916 June 18	19 ^h 00 ^m	4 ^d 095	— 43.0	+0.1	0.5
4985*	Aug. 13	16 33	4.281	— 56.8	— 1.8	0.5
5759*	1917 May 3	21 30	1.815	— 84.5	— 4.8	0.5
5862*	June 9	20 54	0.225	— 67.8	+2.3	0.5
5918*	July 1	18 42	0.708	— 93.8	— 0.2	0.5
6040*	Aug. 2	16 15	2.611	— 36.7	+6.8	0.5
6861*	1918 Apr. 25	23 26	3.240	— 20.8	+2.6	0.5
7231*	Aug. 21	16 18	0.962	— 94.8	+3.3	0.5
C 349	1920 Apr. 3	23 15	0.917	— 99.0	— 1.3	1.0
354	4	23 42	1.936	— 75.3	— 0.8	1.0
359	5	21 28	2.843	— 31.7	+2.4	1.0
363	6	22 26	3.884	— 29.9	+2.2	1.0
407	May 6	20 57	3.826	— 31.0	— 1.3	1.0
413	7	21 32	0.505	— 85.1	+3.5	1.0
415	8	18 45	1.450	— 88.9	+3.5	1.0
433	26	18 57	2.317	— 59.6	— 2.6	1.0
465	June 4	22 14	2.885	— 35.9	— 3.3	1.0
468	5	18 30	3.729	— 26.5	0.0	1.0
471	6	16 10	0.337	— 80.8	— 3.8	1.0
492†	26	21 30	3.427	— 20.6	+1.5	0.5
557	July 28	16 36	0.944	— 101.1	— 3.1	1.0

* 7-inch camera.

† Underexposed.

The first eight plates were secured with a camera of 7 inches focal length instead of the usual 18-inch camera. These and one of the later plates, which was much underexposed, were given half-weight, all the others unit weight. A least-squares solution with twenty-one conditional equations was made by Schlesinger's method for the correction of all six elements. The changes for P , K , and γ were small, while those for the other three elements were of sufficient size to require a second least-squares solution for these three unknowns. The deviations from an ephemeris based on the resulting elements showed satisfactory agreement with the

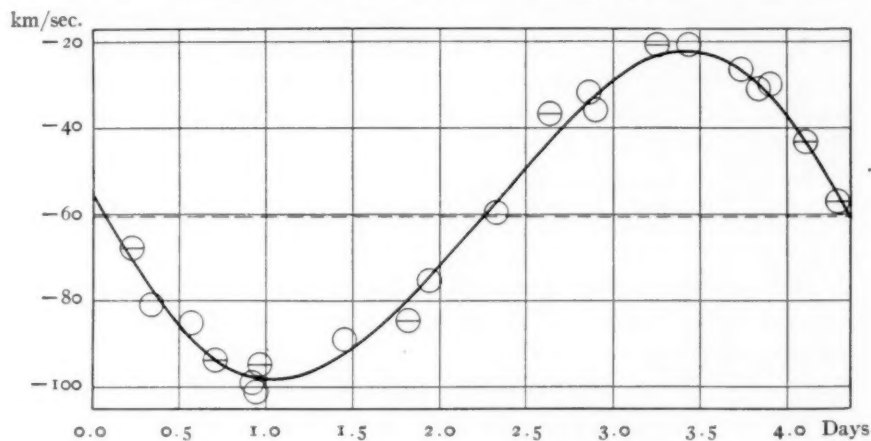


FIG. 3.—Velocity-curve of Lalande 29330

residuals given by the conditional equations. These elements were therefore adopted as final. With their probable errors they are:

$$\begin{aligned}
 P &= 4.28503 \pm 0.00016 \text{ days} & K &= 38.1 \pm 0.3 \text{ km/sec.} \\
 e &= 0.089 \pm 0.024 & T &= \text{J.D. } 2,422,418.0522 \pm 0.0221 \\
 \omega &= 82^\circ 49' \pm 13'.40 & \gamma &= -60.6 \text{ km/sec.} \\
 a \sin i &= 2,238,160 \text{ km} \\
 \frac{m_1^3 \sin^3 i}{(m+m_1)^2} &= 0.0244 \odot
 \end{aligned}$$

The corresponding value of Σpv^2 is one-half that from the preliminary elements.

Reference to the table of observations shows that the first eight spectrograms obtained with low dispersion give satisfactory

residuals, whose mean is very little larger than that of the other residuals. This result tends to give one confidence in the observed velocities of faint stars of late spectral class, which must be obtained with low dispersion. In the velocity diagram (Fig. 3) barred circles represent the eight velocities derived with low dispersion.

It is noteworthy that we have here a star absolutely faint ($M = +6.0$) as the brighter member of a spectroscopic binary whose period is short and whose eccentricity is small for stars of its spectral class. Attention may also be called to the large velocity of the center of mass. The space motion derived from the radial velocity, proper motion, and parallax is 75.8 km/sec. with its apex at $A = 116.7^\circ$, $D = -6.8^\circ$. The rectangular components, x , y , z , where x and y are parallel to the galactic plane, are $x = -72.2$, $y = -16.7$, $z = +12.8$ km/sec.; further $L = 193^\circ$ and $B = 10^\circ$, where L and B are the galactic longitude and latitude of the apex. These may be compared with the mean values found by Adams and Joy¹ for 29 stars with large radial velocities: $\bar{x} = -59.5$, $\bar{y} = -44.5$, $\bar{z} = -4.4$, $\bar{v} = 74.4$ km/sec., $L = 217^\circ$, $B = -3^\circ$.

The agreement with the stars used by Adams and Joy is seen to even better advantage when x , y , z are plotted upon their velocity diagram.

COMPANION TO α HERCULIS

This star, hereafter referred to as B , was found from measures of the first two plates to have a variable radial velocity. It is the companion to α Herculis, hereafter designated A , whose spectral class is Mb and whose magnitude varies between 3.1 and 3.9, with a period that is uncertain if not irregular. In the visual pair no certain evidence of orbital motion has been found. Boss gives a proper motion for A of $0''.030$ and for B of $0''.032$. The magnitude of B is 5.4 and its spectral class F9, forming, with A , a striking visual pair, apparently red and blue. Their separation, $5''$, makes it easily possible to obtain a pure spectrum of the fainter star with either the 60-inch or 100-inch telescope, provided the seeing is good.

Table V contains the data for the twenty-eight spectrograms available for the orbit. A period of 51.6 days gave a satisfactory

¹*Mt. Wilson Contr.*, No. 163, 1919; *Astrophysical Journal*, 49, 179, 1919.

assemblage of the velocities about a single epoch and furnished a velocity-curve from which the following preliminary elements were derived:

$$\begin{aligned} P &= 51.6 \text{ days} & K &= 31.8 \text{ km/sec.} \\ e &= 0.08 & T &= \text{J.D. } 2,422,467.13 \\ \omega &= 15^\circ 5 & \gamma &= -37.2 \text{ km/sec.} \end{aligned}$$

TABLE V
OBSERVATIONS OF THE COMPANION TO α HERCULIS

Plate No.	Date	G.M.T.	Phase	Velocity	O—C
				km/sec.	km/sec.
C 60*....	1919 Sept. 10	15 ^b 23 ^m	2 ^d 010	-9.2	+5.4
66†....	19	15 58	11.034	-43.4	+2.2
114.....	Oct. 12	15 03	33.996	-36.2	+4.1
350.....	1920 Apr. 4	0 48	2.042	-18.6	-3.9
408.....	May 6	22 28	34.947	-33.3	+3.7
416.....	8	21 04	36.887	-29.0	+1.1
422.....	10	23 20	38.980	-25.0	-1.9
430.....	25	23 35	2.400	-18.5	-2.9
442*....	28	23 39	5.405	-28.3	-3.2
470†....	June 5	23 48	13.411	-48.2	+5.0
477‡§....	7	21 10	15.301	-55.8	+2.5
508.....	July 3	15 55	41.083	-17.7	-0.9
510.....	5	19 24	43.227	-10.1	+1.5
527.....	7	16 27	45.104	-16.8	-8.3
539*....	23	15 55	9.493	-45.6	-5.5
79404*....	30	16 13	16.505	-64.9	-4.0
C 573.....	Aug. 2	15 27	19.473	-64.3	+0.9
79431*....	4	17 50	21.572	-68.0	-2.0
C 622.....	30	15 07	47.460	-6.0	-0.1
C 627.....	31	15 08	48.460	-0.8	+6.0
79514*....	Sept. 1	14 58	49.452	-11.4	-4.0
79531.....	3	16 15	51.506	-3.0	+7.5
C 647 	5	15 04	1.867	-13.7	+0.5
C 664.....	25	15 17	21.876	-65.3	+0.7
79573.....	26	15 07	22.870	-67.4	-1.7
79581.....	27	14 42	23.852	-64.2	+0.8
79591.....	28	14 58	24.863	-65.5	-1.6
79616§....	Oct. 3	14 51	29.858	-58.0	-4.5

* Overexposed.

† Focus poor.

‡ Seed 23.

§ Strongly exposed.

|| Somewhat underexposed.

Since the interval between the first and the last observation covers but seven revolutions of the binary in its orbit, it seemed best to derive a correction for the period from the least-squares solution, together with corrections to the other five elements. As the table shows, B , which is really a very bright star for the one-prism spectrograph and 100-inch telescope, has given in several

cases overexposed plates. It is assumed that the necessity of measuring upon such plates farther to the violet than usual has given the advantage of greater dispersion, which compensates for some falling off in focus. Hence all values of the radial velocity were given weight unity, and corrections derived from the set of six normal equations based upon the twenty-eight conditional equations. The resulting corrections proved to be moderate in size and effected a reduction of 25 per cent in the value of Σpv^2 .

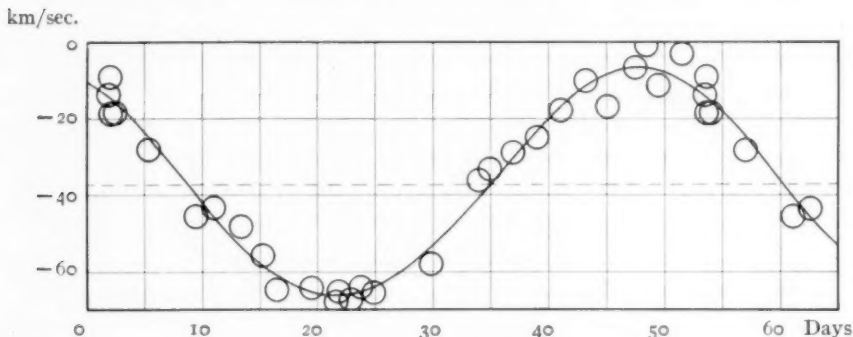


FIG. 4.—Velocity-curve of companion to α Herculis

Hence they have been adopted as final and with their probable errors are to be found below:

$$P = 51.590 \pm 0.002 \text{ days}$$

$$e = 0.028 \pm 0.026$$

$$\omega = 27^\circ 89' \pm 19^\circ 3'$$

$$K = 29.64 \pm 0.77 \text{ km/sec.}$$

$$T = \text{J.D. } 2,422,468.581 \pm 3^d 0.30$$

$$\gamma = -37.2 \text{ km/sec.}$$

$$\frac{m_1^2 \sin^3 i}{(m + m_1)^2} = 0.1395 \odot$$

$$a \sin i = 2,102,700 \text{ km}$$

The probable error derived for a determination of velocity of weight unity is ± 2.85 km/sec.

Attention is called to the fact¹ that the radial velocity of *A* is -32.2 km/sec., whereas that for the center of mass of *B* as here determined is -37.2 km/sec., showing a difference whose reality, in part at least, can hardly be doubted. Taking the distance between *A* and *B* as $5''$, the projected linear separation obtained by using the spectroscopic parallax $0''.018$ derived for *B* turns out to be greater than two hundred and fifty astronomical

¹ *Lick Observatory Bulletin*, No. 229, 1913.

units. The true separation cannot be less than this amount. This, with the considerable difference in the radial velocities, is evidence of a very loose or practically negligible gravitational connection between the two stars unless the masses are very large. It is not improbable that we have here an optical pair.

Coblentz¹ radiometric measures of *B* led Burns² to conclude that light from *A* was probably involved in the measurement. This seems very probable in view of experience with the 100-inch and 60-inch telescopes whose focal lengths, in Cassegrain form, give a much greater linear separation of *A* and *B* in the focal plane than would be the case with the Crossley reflector used in Newtonian form. It may be pointed out, however, that the only indication of spectral class available to Coblentz, viz., the designation as a "blue" star, is not borne out by the spectrum, which is F9, hence the star is approximately "yellow." Although it is certain that the secondary member of the spectroscopic binary is photographically at least one magnitude fainter than the primary, for the spectrum is not recorded, its spectral class may be such as to help account for the discrepancy in Coblentz' measures.

205 DRACONIS

This binary is evidently the brighter component of the visual double β 971. The fainter component is of magnitude 8.5. Burnham says "it, β 971, is certainly a binary and in rapid motion. . . . The measures indicate that the plane of the orbit is nearly in the line of sight. The period will probably be short." The last observation reported by him in which the two stars were resolved is by Aitken:

1905.29 p.a. 26°6 dist. 0".37.

Thirty spectrograms were obtained between August 22, 1918, and November 14, 1919. The data are in Table VI. Both spectra show lines when the difference in velocity is sufficient, but the lines are so poor that considerable uncertainty attaches to the measured velocities, a difficulty increased at times by the blending of lines of the two spectra. Observations upon successive nights considered

¹ *Lick Observatory Bulletin*, No. 266, 1914.

² *Publications of the Astronomical Society of the Pacific*, 27, 110, 1915.

in connection with the results for pairs of plates upon the same night led to a trial of periods around four days. $P=3.76468$ days was found to represent the observations best and was adopted as final. When all the observations had been gathered about a single epoch with this period, the two resulting velocity-curves led to the following preliminary elements:

$$\begin{aligned} P &= 3.76468 \text{ days} & K &= 96.5 \text{ km/sec.} \\ e &= 0.10 & K_1 &= 102.5 \text{ km/sec.} \\ \omega &= 166^\circ & T &= \text{J.D. } 2,422,161.597 \\ \omega_1 &= 346^\circ & \gamma &= -14.0 \text{ km/sec.} \end{aligned}$$

TABLE VI
OBSERVATIONS OF 205 DRACONIS

PLATE NO.	DATE	G.M.T.	PHASE	VELOCITY		O-C	
				Primary	Secondary	Primary	Secondary
				km/sec.		km/sec.	
γ 7239...	1918 Aug. 22	18 ^h 37 ^m	0 ^d 200	+ 83.9	-112.4	+16.9	- 7.3
7376...	Sept. 20	18 41	2.948	-23.3		(-24.7)	(+15.8)
7385...	21	17 50	0.149	+ 87.0	-116.0	+11.1	- 1.9
7395...	22	17 50	1.149	-15.1		(+36.9)	(-29.7)
7404...	23	17 01	2.115	-109.2	+ 77.1	0.0	+ 4.9
7523...	Oct. 26	15 05	1.154	-25.8		(+27.0)	(-41.2)
8328*	1919 July 3	20 22	2.904	+ 28.7	- 53.7	(+34.4)	(-21.7)
8354...	9	23 17	1.495	- 83.6	+ 56.4	+13.2	- 3.3
8374...	13	18 22	1.525	-109.1	+ 79.4	- 9.5	+16.9
8411...	17	16 57	1.702	-111.0	+ 72.8	+ 1.1	- 2.3
8416...	17	23 39	1.981	-122.9	+ 88.7	- 7.7	+10.5
8417†...	18	16 50	2.697	- 68.1	+ 23.4	(-28.8)	(+21.6)
8435...	19	16 45	3.694	+ 60.6	-111.3	-17.6	+ 5.1
8438...	19	21 36	0.131	+ 83.9	-117.9	+ 7.3	- 3.1
8441...	Aug. 2	16 27	2.622	-27.2		(+23.8)	(-40.8)
8444...	2	21 22	2.827	-21.7		(- 3.5)	(- 2.3)
8447...	3	16 14	3.615	+ 77.4	-110.4	+ 1.5	+ 3.6
8458‡...	4	16 10	0.852	+ 15.7	- 56.0	(+20.0)	(-22.6)
8462...	4	21 55	1.085	-14.0		(+28.0)	(- 9.4)
8467...	5	15 55	1.835	-103.5	+ 77.1	+12.7	- 2.1
8471...	5	21 40	2.075	-117.9	+ 76.0	- 6.4	+ 1.5
8703...	Sept. 12	15 35	2.172	- 93.3	+ 42.7	+12.0	-25.5
8765§...	Oct. 11	15 13	1.042	- 51.5	+ 16.7	(-16.3)	(+19.0)
8776...	12	15 23	2.049	-111.3	+ 66.5	+ 1.4	- 9.2
C 118...	13	15 28	3.052	+ 29.8	- 55.7	+12.2	- 0.3
γ 8786...	14	15 18	0.281	+ 63.7	-113.3	- 4.7	- 6.8
C 156 ...	Nov. 1	15 55	3.248	+ 35.3	- 83.5	- 9.5	- 0.7
171...	4	15 44	2.476	- 91.0	+ 44.2	-18.7	+ 9.1
210 ...	13	15 10	0.158	+ 76.5	-118.0	+ 1.0	- 4.3
215¶...	14	14 35	1.134	- 50.9	+ 15.2	(- 1.2)	(+ 2.9)

* First measured as single spectrum; vel. -10.1 km/sec.

† First measured as single spectrum; vel. -11.9 km/sec.

‡ First measured as single spectrum; vel. -22.0 km/sec.

§ First measured as single spectrum; vel. -15.6 km/sec.; Seed 23.

¶ First measured as single spectrum; vel. -18.6 km/sec.; Seed 23.

|| Seed 23.

In deriving the corrections to the elements by the method of least squares only those spectrograms were used, nineteen in all, upon which both spectra had been measured. These were given equal weight and combined into twelve normal places which resulted in twenty-four conditional equations of the form given by Harper.

The correction found for e reduced it to zero, thus giving a circular orbit in which of course ω and T are arbitrary. The time of maximum positive velocity for the primary was adopted for T .

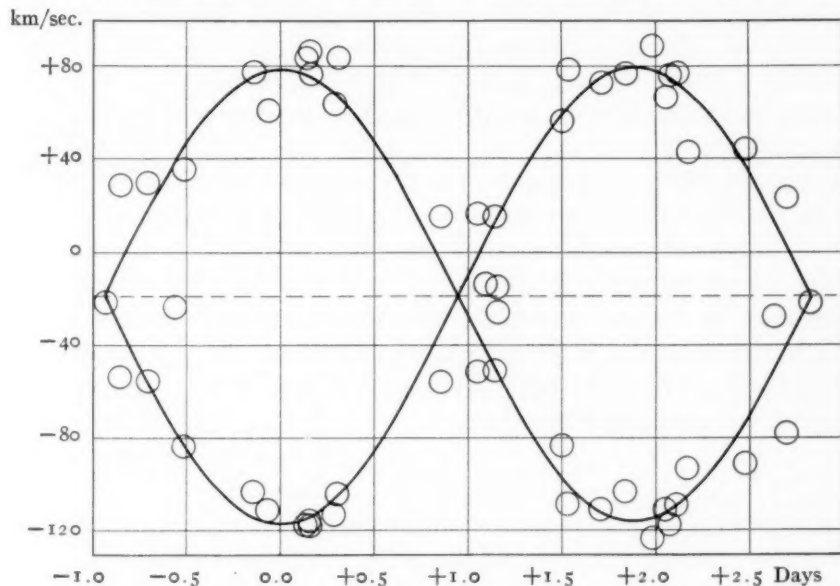


FIG. 5.—Velocity-curves of 205 Draconis

Since some of the corrections were rather large, a second least-squares solution, based upon the circular elements, was made. The quantity, Σpv^2 , for the final elements is about 25 per cent of the value it had for the preliminary elements. The preliminary elements of this binary are somewhat less accurate than was the case for the others. Corrected as indicated by the two solutions they gave the following elements and probable errors:

$$P = 3.76468 \text{ days}$$

$$K = 97.7 \pm 2.0 \text{ km/sec.}$$

$$K_1 = 98.3 \pm 2.0 \text{ km/sec.}$$

$$T = \text{J.D. } 2,422,159.7691 \pm 0.0178$$

$$\gamma = -18.8 \text{ km/sec.}$$

$$a \sin i = 5,058,000 \text{ km}$$

$$a_1 \sin i = 5,089,000 \text{ km}$$

$$m \sin^3 i = 1.48 \odot$$

$$m_1 \sin^3 i = 1.47 \odot$$

The probable error of a velocity of unit weight derived from the nineteen spectrograms whose spectra are separated is ± 4.5 km/sec. In the table of observations the residuals for those plates which did not contribute to the normal places have been placed in parentheses. It will be seen from the velocities or from the remarks that in all cases they were first measured as single spectra. Later it was found that by going to the extreme violet end of the spectrum the increased dispersion gave some material for the velocities of both components, but in a region that has already departed from good focus. These measures have been given in the velocity columns, but the residuals indicate their relative inferiority.

The intensities of the absorption lines for the two components were so little different that they could not be used as a reliable criterion of phase. The results for K and K_1 and the similarity of the spectra are in accord with this lack of difference in intensity.

If Burnham's conclusion that the plane of the orbit of the visual double is nearly in the line of sight is correct, and if the inclination of the spectroscopic orbit should be nearly the same, this binary would be a favorable case to examine for variability caused by eclipse. The large amplitudes K and K_1 are likewise favorable to this possibility.

BOSS 5591

Table VII contains the data for the nineteen spectrograms which form the material for deriving the orbit. Sixteen of these gave velocities for both components. The phase could not be determined with certainty from differences in the intensities of the spectral lines of the two components, which seem to be identical in spectral class A9. As in the case of 205 Draconis, plates on successive nights, combined with pairs upon the same night (the latter to be sure that the period was not less than a day), resulted in a choice of 3.74860 days as best for grouping the observations about a single epoch. Preliminary elements were obtained as follows:

$P = 3.74860$ days	$K = 97.8$ km/sec.
$e = 0.175$	$K_1 = 98.0$ km/sec.
$\omega = 89^\circ.2$	$T = \text{J.D. } 2,422,175.243$
$\omega_1 = 269^\circ.2$	$\gamma = +0.3$ km/sec.

In correcting the preliminary elements only the velocities from plates showing both spectra were used, sixteen in all. The period was assumed to need no correction. Three pairs of plates, taken in each case at nearly the same time and at favorable parts of the orbit, furnished three normal places. Otherwise each plate furnished a conditional equation for both primary and secondary star. All plates used were given unit weight.

Although corrections to the preliminary elements were twice derived by the method of least squares applied to the twenty-six

TABLE VII
OBSERVATIONS OF BOSS 5591

PLATE NO.	DATE	G.M.T.	PHASE	VELOCITY		O-C	
				Second.	Prim.	Second.	Prim.
				km/sec.		km/sec.	
γ 7408.....	1918 Sept. 23	19 ^h 11 ^m	0 ^d 524	- 63.4	+65.7	+12.9	-17.7
7577.....	Nov. 21	15 10	3.128	+ 94.7	-81.0	- 5.1	+ 9.1
7594.....	22	16 14	0.423	- 81.3	+81.4	-15.9	+ 8.7
7626.....	Dec. 13	15 16	2.641	+ 68.4	-77.4	- 6.4	-11.9
7636.....	14	14 55	3.627	+ 31.8	-33.9	-13.8	+ 2.8
7643.....	15	15 02	0.882	- 98.2	+97.8	-12.3	+ 4.9
8365.....	1919 July 11	23 14	3.051	+104.3	-86.4	+ 5.1	+ 3.2
8376.....	13	21 55	1.247	- 74.2	+76.0	- 8.2	+ 2.6
8413.....	17	20 38	1.445	- 50.0	+65.6	- 1.0	+ 8.9
8437.....	19	19 45	3.408	+ 94.5	-82.0	+11.9	- 8.9
8443.....	Aug. 2	19 57	2.421	+ 0.2			
8451.....	3	19 50	3.417	+ 87.1	-85.8	+ 5.7	-13.8
8456.....	4	0 05	3.594	+ 59.9	-52.5	+ 7.7	- 9.2
8460.....	4	20 06	0.680	- 76.3	+95.6	+ 9.1	+ 3.2
8469.....	5	18 40	1.620	+ 5.0			
8473.....	6	0 12	1.859	- 0.4			
C 119*.....	Oct. 13	16 22	3.041	+105.4	-80.0	+ 6.2	+ 9.5
123*.....	14	16 56	0.326	- 50.8	+66.3	+ 0.2	+ 7.8
129*.....	15	16 46	1.316	- 49.3	+60.7	+11.3	- 7.2

* Seed 23.

conditional equations, the corrections from the second solution were so small that the elements adopted as final and their probable errors are based upon the results of the first solution. These are:

$$\begin{aligned}
 P &= 3.74860 \text{ days} & T &= \text{J.D. } 2,422,175.1577 \pm 0^d 0880 \\
 e &= 0.189 \pm 0.021 & \gamma &= +4.2 \text{ km/sec.} \\
 \omega_1 &= 82^\circ 66' \pm 9'' 00 & a_1 \sin i &= 4,728,000 \text{ km} \\
 \omega &= 262^\circ 66' \pm 9'' 00 & a \sin i &= 4,662,000 \text{ km} \\
 K_1 &= 93.2 \pm 2.8 \text{ km/sec.} & m_1 \sin^3 i &= 1.17 \odot \\
 K &= 92.1 \pm 2.8 \text{ km/sec.} & m \sin^3 i &= 1.19 \odot
 \end{aligned}$$

Attention is called to the fact that in giving the final elements, the order is the same as for the preliminary elements, but the subscripts have been changed, because the corrections to the elements have interchanged the two stars in respect to the semi-amplitudes, K and K_1 , and hence to their masses.

Substitution of the unknowns derived from the first solution in the conditional equations gives residuals in substantial agreement with those derived from the final elements. Σpv^2 is 64 per

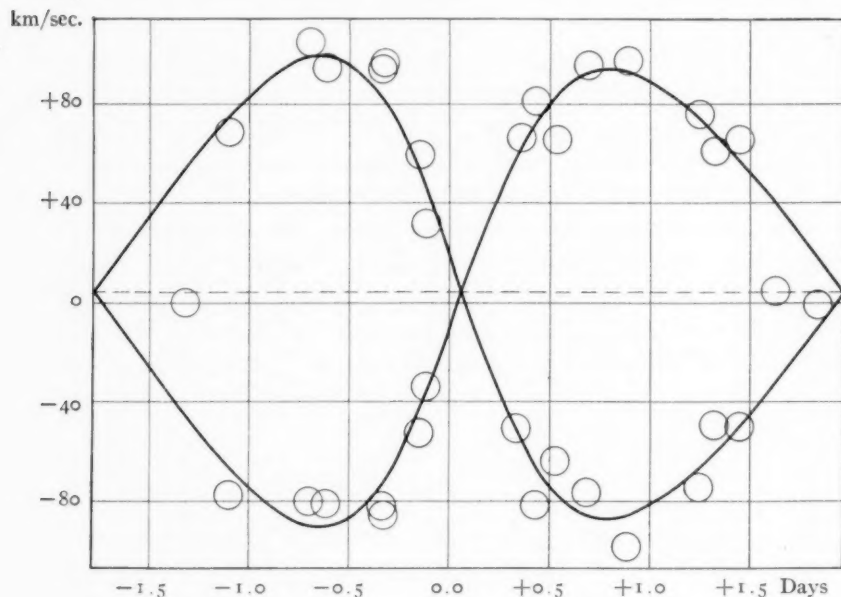


FIG. 6.—Velocity-curves of Boss 5591

cent of its value for the preliminary elements. The probable error of a velocity of unit weight for either component, on the basis of all residuals for plates with double spectra, is ± 6.4 km/sec. This is larger than that for either of the other two spectroscopic binaries showing double lines, and is to be accounted for in part by the fact that all plates were given unit weight. The spectral class is somewhat earlier than that of the others, which involves a greater uncertainty in the measures.

LALANDE 46867

This spectroscopic binary shows only the spectrum of a single star of class K2 from which the absolute magnitude derived is +6.0. The twenty-five spectrograms available are listed in Table VIII. The velocity-curve, which proved satisfactory when all observations were assembled around a single epoch with $P=6.7217$ days, yielded the following preliminary results:

$$\begin{array}{ll} P=6.7217 \text{ days} & K=+39.2 \text{ km/sec.} \\ e=0.08 & T=\text{J.D. } 2,422,220.768 \\ \omega=20^{\circ}5 & \gamma=-20.2 \text{ km/sec.} \end{array}$$

Plate 79492, for which the preliminary elements give a residual far larger than for any other plate in the table and which was found to be in poor focus except in the region of the H and K lines, was omitted from the least-squares solution but retained in the table because the velocity derived from the emission lines H and K, to be referred to later, agrees fairly well with the velocity computed from the elements. The twenty-four remaining plates were given equal weight and the period was assumed to be known with the requisite accuracy, since the interval between the first and last spectrograms corresponds to one hundred and seventy-nine revolutions of the binary in its orbit. When the remaining five elements had been corrected by the method of least squares, the new elements were found to decrease the quantity $\Sigma p v^2$ by only 3 per cent. Therefore, as so corrected, they are adopted as final and are given below with their probable errors:

$$\begin{array}{ll} P=6.7217 \text{ days} & \gamma=-19.8 \text{ km/sec.} \\ e=0.059 \pm 0.019 & \frac{m_1^3 \sin^3 i}{(m+m_1)^2} = 0.0125 \odot \\ \omega=18^{\circ}6 \pm 16^{\circ}3 & a \sin i = 3,552,300 \text{ km} \\ K=38.5 \pm 0.76 \text{ km/sec.} & \\ T=\text{J.D. } 2,422,220.7403 \pm 0^d 3005 & \end{array}$$

The probable error for a single determination of velocity of weight unity is ± 2.8 km/sec.

Plate 79671, obtained by Mr. Hoge on a partly cloudy night, is very strongly exposed and was noticed by him to have two emission lines in the violet. These proved to be H and K of calcium superposed upon their usual broad absorption lines. Thereafter an effort was made to expose plates strongly enough to show this

feature of the spectrum. By using earlier plates which upon second examination showed this feature, velocities from these two emission lines were secured from twelve plates in all. They are listed in a separate column in Table VIII and placed in parentheses whenever they are derived from plates on which these lines are extremely feeble. Judging from the better-exposed plates, the

TABLE VIII
OBSERVATIONS OF LALANDE 46867

Plate No.	Date	G.M.T.	Phase	Velocity Abs. Lines	Velocity H and K	O-C Abs. Lines
				km/sec.	km/sec.	km/sec.
$\gamma 6036$	1917 July 31	23 ^h 28 ^m	0 ^d 016	-11.1	-3.3
6367....	Nov. 23	16 38	1.401	-25.5	-1.0
6499....	Dec. 29	15 01	3.725	-46.5	+2.3
7214*....	1918 Aug. 17	23 39	6.548	+28.5	+7.9
7271....	27	22 47	3.067	-55.0	+1.0
7410....	Sept. 23	21 10	3.114	-53.0	+2.9
8722*....	1919 Sept. 14	22 40	2.927	-60.5	-4.4
C 555....	1920 July 27	20 36	3.921	-49.7	-5.0
$\gamma 9434$	Aug. 4	22 33	5.278	-4.2	-2.6
9492†....	28	21 18	2.340	-36.7	-46.4	+14.0
9504....	29	23 25	3.440	-48.6	(-62.4)	+4.6
C 626....	31	0 13	4.463	-33.5	-4.0
C 630....	31	23 10	5.418	+5.9	+2.9
$\gamma 9518$ †....	Sept. 1	21 26	6.357	+15.4	+13.3	-5.5
C 665....	25	16 27	3.252	-58.1	-3.0
$\gamma 9578$	26	20 21	4.415	-27.9	+3.1
9587....	27	21 00	5.442	+8.6	(+6.0)	+4.9
9595....	28	19 43	6.389	+20.6	(+25.9)	-0.3
9671....	Oct. 26	19 01	0.750	+4.2	+2.1	+4.5
9678....	27	20 15	1.802	-39.4	-29.6	-1.5
9680....	28	16 45	2.656	-55.0	-57.7	-0.2
C 738....	31	19 31	5.772	+9.7	+9.3	-3.2
C 743....	Nov. 1	17 32	6.689	+18.2	+7.7	-1.1
$\gamma 9734$	21	16 25	6.478	+18.8	+20.2	-2.1
C 774....	23	19 19	1.876	-35.8	(-34.6)	+4.2

* Underexposed.

† Best focus around H and K. Plate not used in least-squares solution.

‡ Focus poor.

H and K emission lines yield the same velocities as the lines of the absorption spectrum. No satisfactory measures of the H and K absorption lines could be made.

In general the absorption lines of Lalande 46867 seem to be less sharp than for the general run of stars of spectral class K2.

The spectroscopic binary σ Geminorum, spectral class G9, abs. mag. +1.1,¹ has been noted both by H. M. Reese,² who announced

¹ *Mt. Wilson Contr.*, No. 142; *Astrophysical Journal*, 46, 313, 1917.

² *Lick Observatory Bulletin*, No. 31, 2, 29, 1903.

its variable velocity, and by Harper,¹ who published its orbit, to have abnormally fuzzy lines. Schwarzschild² reports having observed H and K emission lines in objective-prism spectrograms of σ Geminorum. A plate of this star obtained by Mr. Duncan for the writer showed emission lines H and K but, relatively to the adjacent continuous spectrum, these were weaker than in Lalande 46867. The velocity derived from the emission lines was in close agreement with that for the absorption lines and agreed as well as

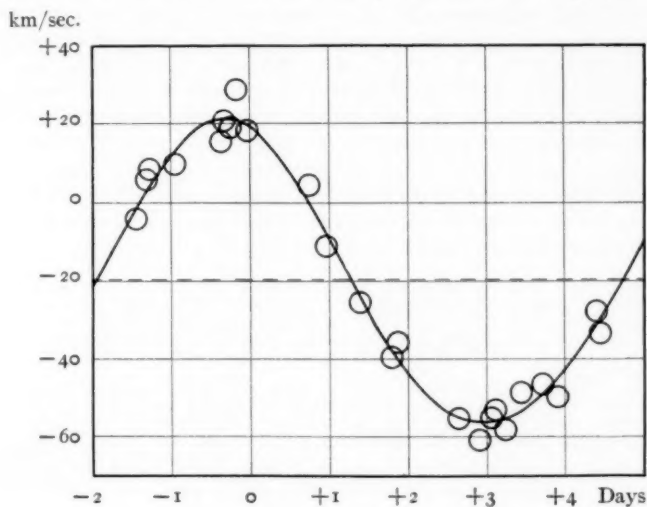


FIG. 7.—Velocity-curve of Lalande 46867

could be expected with the velocity computed from Harper's elements. Hence these stars present several points of similarity: (a) spectral class; (b) character of absorption; (c) presence of emission lines for H and K; and (d) the fact that both are spectroscopic binaries with eccentricities smaller and periods shorter than the average for binaries of this spectral class. There is a notable difference in the absolute magnitude (σ Geminorum, +1.1, Lalande 46867, +6.0), associated with emission lines which are stronger relatively to the continuous spectrum in the case of the absolutely fainter star.

MOUNT WILSON OBSERVATORY
October 1920

¹ *Publications of the Royal Astronomical Society of Canada*, 5, 200, 1911.

² *Astrophysical Journal*, 38, 292, 1913.

PRESSURE-SHIFTS IN A CALCIUM ARC

By L. F. MILLER

ABSTRACT

Pressure-shifts in calcium-arc spectrum, λ 3150 to λ 6500.—A 4-mm, 4-amp. arc between Ca electrodes, one water-cooled, the other pointed, was operated in a chamber in which the pressure was varied from 5 to 76 cm of Hg. By using the second order of a 21.5-ft. grating, shifts of 0.001 Å could be detected. Table II gives the mean results for 75 lines. The H and K lines, the P and T series lines, and a few other lines show a shift of 0.001 Å or less. All lines which broaden toward the violet show a shift toward the violet; all others tend to shift toward the red. The shift is the same for all lines of each term of a series, and increases with the term number, except that there is an indication that for the lower term of the T₁ series the shift is positive instead of negative as in the case of terms 4 to 6. Mechanical shifts were eliminated by comparison with iron lines of class *a* and *b*.

The present paper gives results on the measurement of pressure-shifts for all the principal lines in the photographic region of the calcium spectrum.

Apparatus.—It will not be necessary to write a detailed description of the apparatus, as conditions were kept, as far as possible, similar to those described by Gale and Whitney¹ in their study of the pole-effect.

Light source.—The arc was formed between two calcium electrodes 9 mm in diameter, arranged in a horizontal position within a closed steel chamber where the pressure could be varied by means of an ordinary oil pump. The pressure was changed from about 5 cm mercury as the lower pressure to atmospheric pressure as the higher pressure. If a greater value had been used, many of the most important lines, especially those of the first and second subordinate "triplet" series, would have become too broadened and diffuse for measurement. It was endeavored to maintain a constant length of arc of 4 mm. The positive pole was water-cooled and flat-faced, while the negative pole was rather sharply pointed. By this arrangement it was found possible to maintain a

¹ *Astrophysical Journal*, 44, 65, 1916.

fairly constant arc. Interruption would occur principally when, after operating for a time, the electrode surface became coated with a crusted formation of non-conducting oxide. This length of time depended more or less on the amount of air present and the success with which the action of the arc could be uniformly distributed over the surface of the electrode by the hand regulation of the arc. On some of the longer exposures it was necessary to open up the chamber one or more times during the course of the exposure to file off and clean the electrodes. The latter were sawed and machined out of very irregular chunks. The metal did not always seem to be of uniform consistency, and apparently this caused at times much fluctuation of the arc and rapid deterioration of the surface.

Source of current.—A 110-volt direct current was used to operate the arc. A suitable resistance was placed in the circuit to maintain a current of about 4 amperes.

Condensing system.—The quartz window in the pressure chamber and the quartz condensing lens made it possible to obtain all the desired lines between λ 6499.8 and λ 3158.9. By means of convenient regulations on the pressure chamber and condensing lens, the middle point of the arc image, enlarged four times, could be readily maintained on the slit.

Dispersing system.—The second-order spectrum of a 21.5-foot Rowland concave grating was employed. The scale was 1.32 Å per mm.

Photographs.—The size of photographic plate used was 2×19 inches, a length great enough to cover about 600 Å.

A large part of the work presented in this report was performed about three years ago, when it was almost impossible to get suitable materials. The war cut off completely the supply of extra-thin glass which had previously been used for these plates and which fitted so well the curvature of the plate-holder. The thinnest glass obtainable at this time was between $1/16$ inch and $1/8$ inch. Some of the plates could only be made to bend along the arc without breaking by slightly warming them and allowing each plate to rest on end supports with a suitable weight on the middle for a number of hours. This seemed to loosen up the glass structure

sufficiently to permit safely the necessary bending in the plate-holder. Films were used at the red end of the spectrum, but since the plate-holder was not made for films it was difficult to make them lie smoothly. If heavier celluloid could have been obtained, the films would have been very satisfactory. The occulting device described and used by Whitney¹ was employed in this work. Being mounted from the floor, it avoided any danger of mechanical shifts due to changing of the shutter. The latter was arranged so as to expose a middle strip upon the plate when in one position and a strip above and below when in the other position. The middle was generally exposed at the high pressure and the strip above and below at the low pressure.

Method of measurement.—The measurements of the shifts were obtained with a small comparator made by Wm. Gaertner & Co. A small candle-power incandescent lamp was arranged with a condensing lens to illuminate the field. This permitted a large increase of illumination of the field when necessary, with very little increase of the intensity of light about the room. A rheostat in circuit also permitted the adjustment of the intensity of illumination to any desirable degree.

It is well known by those working in this field that, because of the varying character of lines as well as incomplete and irregular photographic action, it is often necessary to study lines to a certain extent in order to determine their character, and that, on the other hand, looking at a line too long will fatigue the eye. Interposing a piece of green glass to give a green-colored field, especially with the more intense illuminations, was found of the greatest aid in reducing this fatigue.

Procedure.—In order to obtain a measure of any mechanical shift which might occur during any exposure, iron filings were imbedded in, or placed on the top edge of, the electrodes, and from the pressure-shifts of certain iron lines previously determined by Gale and Adams² the correction constant was determined. All mechanical shifts were measured by those iron lines which are classified by Gale and Adams³ as class *a* lines. In a few instances

¹ *Astrophysical Journal*, 44, 65, 1916.

² *Ibid.*, 35, 10, 1912; 37, 391, 1913.

³ *Ibid.*

some class *b* lines were used as a double check on the mechanical shift, and in one instance a class *c* line. The measurements with these latter lines corroborated those of the *a* lines. Exposures of the whole photographic region were obtained by moving the camera to seven successive positions from the red to the violet, as will be observed in Table I. This allowed considerable overlapping of some regions, so that the correction constant from the iron lines could be applied to two adjacent regions, and afforded a double check on the mechanical shift.

TABLE I

	λ	λ
Region A.....	6500	to 5900
Region P.....	6150	to 5550
Region B.....	5857	to 5261
Region I.....	5270	to 4675
Region C.....	4725	to 4125
Region D.....	4175	to 3575
Region E.....	3725	to 3125

Exposures in regions A and P were made on Wratten and Wainwright "Panchromatic" films; in regions B, I, and C on Cramer "Inst. Iso"; and in regions D and E partly on Cramer "Crown" plates and partly on Seed 27 dry plates. Owing to the difficulty of obtaining sufficient quantities of plates, three or four exposures were usually made on one plate. Incomplete and irregular photographic action on many plates, even on the more intense exposures, seemed to indicate a poor grade of films. Difficulties were also experienced with faulty developers.

Results.—Table II gives the results obtained. All lines are grouped under two general divisions, the series and the non-series groups. The first column indicates the wave-lengths of the lines, and series designation, if any. The second column gives the shifts; the third, the number of plates measured; while the fourth column contains the average difference from the mean. The fifth shows the character of the broadening of the lines, as far as one could judge with a change of pressure of only one atmosphere. Unless there was a fairly definite indication of how the line broadens, it was marked "s," symmetrical. Only a few of the lines seemed

TABLE II*

λ	Shift	Number	Average Difference	Broadening	λ	Shift	Number	Average Difference	Broadening
SERIES GROUP									
$P \begin{cases} 6409.8. \\ 6493.9. \\ 6449.9. \\ 4302.6. \\ 4283.1. \end{cases}$	$\begin{cases} .002 \\ .001 \\ .001 \\ .000 \\ -.001 \end{cases}$	$\begin{cases} 3 \\ 4 \\ 4 \\ 4 \\ 4 \end{cases}$	$\begin{cases} .001 \\ .001 \\ .001 \\ .001 \\ .001 \end{cases}$	$\begin{cases} s \\ s \\ s \\ s \\ s \end{cases}$	$T_{14} \begin{cases} 4456.1 \\ 4454.9 \\ 4435.8 \\ 4435.1 \\ 4425.6 \end{cases}$	$-.001$	23	.001	uv
$SL_2 \begin{cases} 5041.0. \\ 4527.1. \\ 4240.5^\dagger \end{cases}$	$\begin{cases} .015 \\ .027 \\ .027 \end{cases}$	$\begin{cases} 4 \\ 11 \\ 11 \end{cases}$	$\begin{cases} .001 \\ .002 \\ .002 \end{cases}$	$\begin{cases} ur \\ ur \\ ur \end{cases}$	$T_{15} \begin{cases} 3644.8 \\ 3644.5 \\ 3631.1 \\ 3630.8 \\ 3624.1 \end{cases}$	$-.020$	24	.002	uv
$SL_3 \begin{cases} 4878.3. \\ 4355.4. \end{cases}$	$\begin{cases} .031 \\ .068 \end{cases}$	$\begin{cases} 7 \\ 6 \end{cases}$	$\begin{cases} .003 \\ .010 \end{cases}$	$\begin{cases} ur \\ ur \end{cases}$					
$t_4 \begin{cases} 4586.1 \\ 4581.6 \\ 4578.8 \end{cases}$	$.030$	21	.002	ur	$T_{16} \begin{cases} 3361.9 \\ 3350.2 \\ 3344.4 \end{cases}$	$-.054$	12	.008	uv
$t_5 \begin{cases} 4098.6 \\ 4095.0 \\ 4092.7 \end{cases}$	$.044$	16	.003	ur	$T_{23} \begin{cases} 6162.4 \\ 6122.4 \\ 6102.9 \end{cases}$	$.012$	19	.002	s
$T \begin{cases} 4318.8. \\ 4299.1. \\ 4289.5. \end{cases}$	$\begin{cases} -.001 \\ -.001 \\ .000 \end{cases}$	$\begin{cases} 4 \\ 4 \\ 4 \end{cases}$	$\begin{cases} .001 \\ .001 \\ .001 \end{cases}$	$\begin{cases} uv \\ uv \\ uv \end{cases}$	$T_{24} \begin{cases} 3973.8 \\ 3957.2 \\ 3949.1 \end{cases}$	$.016$	18	.003	ur
$P_2 \begin{cases} 3737.1 \\ 3706.1 \end{cases}$	$.008$	14	.001	sh	$T_{25} \begin{cases} 3487.7 \\ 3474.9 \\ 3468.6 \end{cases}$	$.041$	11	.005	ur
$P_1 \begin{cases} 3181.4. \\ 3179.4. \\ 3158.9. \end{cases}$	$\begin{cases} .013 \\ .013 \\ .014 \end{cases}$	$\begin{cases} 5 \\ 4 \\ 3 \end{cases}$	$\begin{cases} .001 \\ .001 \\ .001 \end{cases}$	$\begin{cases} ur \\ ur \\ ur \end{cases}$	$T_{26} \begin{cases} 3286.2 \\ 3274.8 \\ 3269.3 \end{cases}$	$.046$	6	.007	ur
$H \ 3968.6.$.001	8	.001	Rs					
$K \ 3933.8.$.001	8	.001	Rs					
NON-SERIES GROUP									
6471.8.	.002	4	.001	s	5513.1.	$-.023$	6	.003	uv
6462.7.	.001	4	.001	s	5349.6.	.005	7	.001	sh
6439.3.	.001	4	.001	s	5270.4.	.004	5	.001	sh
6169.8.	.012	4	.002	s	5265.7.	.003	9	.001	sh
6169.3.	.013	3	.002	s	5264.4.	.010	6	.001	sh
5857.7.	.022	5	.002	ur	5262.4.	.010	5	.001	sh
5603.1.	.006	12	.002	s	5261.9.	.011	6	.001	sh
5601.1.	.006	12	.001	s	5189.0.	$-.003$	7	.001	uv
5598.6.	.003	4	.001	s	4685.4.	$-.009$	6	.001	uv
5594.6.	.007	4	.002	s	4307.9.	$-.001$	4	.001	uv
5590.3.	.005	10	.001	s	4240.5.				
5588.9.	.005	5	.003	s	4226.9.	.018	7	.002	Rs
5582.1.	.005	10	.001	s					

* The wave-lengths and series designations are from Kayser's *Handbuch*, for ease of comparison with Whitney's tables for pole-effect for these same lines. See Gale and Miller, *Physical Review*, 17, 428, 1921.

† λ 4240.5 was masked by λ 4226.9.

to retain their sharpness more than the others and these are marked "sh." Unsymmetrical broadening to the red or to the violet is designated respectively by "ur" and "uv." Only three lines are reversed with increase of pressure, and they remained symmetrical. These are marked "Rs."

The results were arranged at first according to the order of the wave-lengths of the lines measured. Although there seems to be in general an increase in the magnitude of the pressure-shifts as one proceeds toward the shorter wave-lengths, there are so many exceptions that we cannot as yet see any definite law. The same thing is true in regard to the broadening of the lines.

It is only when the results are arranged according to the series grouping of the lines that the observations take on a more definite character. Royds,¹ in his discussion on the different character of spectrum lines belonging to the same series, raised the question whether calcium would act like iron in that the unsymmetrical widening and pressure-shift both take place in the same direction. It will be observed from Table II that the first subordinate "triplet" series in calcium is unsymmetrically broadened toward the violet and the pressure-shift is toward the violet. In the second subordinate "triplet" series and also in the Fowler series t_4 and t_5 the unsymmetrical broadening is toward the red and the pressure-shift is toward the red.

Royds also raised the question, as suggested by Moore,² whether the Fowler series would have practically zero shift because the Zeeman effect was approximately zero, or whether there would be a large shift because the lines of this series are susceptible to density effects. The latter seems to be the case.

Royds³ also points out in his paper that lines in the same series may have entirely opposite characteristics in one part compared with the lines in another part. Thus in the first subordinate "triplet" series of barium, lines 5819, 5800, 5777, 5536, 5519, and 5424 broaden and shift to the red, while lines 4493, 4489, 4393, 4323, and 4264 broaden and shift to the violet. The same change of characteristics appears possible in the first and second

¹ *Astrophysical Journal*, **41**, 154, 1915.

² *Ibid.*, **33**, 385, 1911.

³ *Ibid.*, **41**, 154, 1915.

subordinate "triplet" series of calcium. As one proceeds from the shorter wave-lengths to the longer, the broadening and shifts decrease, and if these quantities could be measured for the lower members in the infra-red, it appears that they would be in the opposite direction. There are strong indications that this is true at least in the first subordinate series.

There are only a few lines which seem to tend to retain their sharpness more than the others as the pressure increases; they are the lines from $\lambda 5270.4$ to $\lambda 5261.9$ and the lines $\lambda 3737.1$ and $\lambda 3706.1$ (P_2 series).

Only three lines showed reversals in this work, the H and K lines and $\lambda 4226.9$. There are many lines marked "s" which would undoubtedly show unsymmetrical broadening with greater increase of pressure, but it was not perceptible within the limits here used.

I wish to express my appreciation to the members of the Department of Physics of the University of Chicago for their aid and interest, especially Dr. Gale, who suggested this problem. I am also very grateful to the Department of Physics of the University of Nebraska for the loan of their comparator, with which most of the measurements were made.

UNIVERSITY OF CHICAGO
September 1920

PREPARATION OF ABSTRACTS

Every article in the *Astrophysical Journal*, however short, is to be preceded by an abstract which should be submitted by the author with the manuscript. In order that the abstract may aid the reader by furnishing an index and brief summary of the contents of the article, and may also be suitable for reprinting in an abstract journal, it should be of the type of those which have been appearing in the *Astrophysical Journal* during 1920. It is requested that the abstracts be prepared in accordance with the following:

DIRECTIONS AND RULES

1. *Notes*.—Read the article carefully, taking rough notes covering all the new information reported, keeping a specially sharp lookout for new incidental results and suggestions not directly related to the main subject.

Material not new need not be described; a valuable summary of previous work, however, should be noted.

2. *Subtitles*.—Write, first, a title describing the group of results forming the main contribution of the article, including all that belong together. If there are in addition results which do not come under that title, gather them into as few groups as possible and formulate a complete and precise title for each. For examples of such subtitles see the italicized parts of the abstracts for 1920.

Each subtitle should describe the corresponding information so precisely that the chance of any investigator being misled into thinking the article contains the particular information he desires when it does not, or vice versa, may be small. "Zeeman effect for metallic furnace spectra" is too broad unless all metals have been studied, for an investigator may be interested at the time, in only one metal; but "Infra-red arc spectrum of iron to 3μ " evidently satisfies this rule.

In general a subtitle is sufficiently precise if it carries the classification of the information three stages or the equivalent, for instance if it gives (a) the elements and substances, (b) the property, and (c) the phase or range studied.

The subtitles should together form a complete index of the new information; that is they should include every measurement, observation, method, suggestion, and theory which is presented as new and of value in itself. They should be complete in themselves and independent of the main title of the article.

3. *Text*.—Write a paragraph summarizing the main group of results and including the corresponding subtitle either all at the beginning or with parts scattered through the text; and then do likewise for the other groups.

¹The rules and illustrative abstracts were prepared by G. S. Fulcher, chiefly while associated with the National Research Council.

A separate paragraph should be used for each distinct subject involved, but no more than necessary. All material which can easily be grouped together under a single title should be summarized in the same paragraph. Parts of subtitles may be scattered through the text, but the subject of each paragraph must be given at the beginning. Italicize subtitles but no other words or phrases.

The text should summarize the author's conclusions and should transcribe all numerical results of general interest, including all that might be looked for in a table of astronomical and physical constants, with an indication of the accuracy of each. It should give all the information that anyone, not a specialist in the particular field involved, might care to have in his notebook.

Complete sentences should be used except in the case of subtitles. The abstract should be made as readable as the necessary brevity will permit.

4. *Final checking.*—Re-read the article so as to check the abstract and correct any omissions and mistakes; read the subtitles by themselves to see that they properly index the information; and read the abstract to see whether it cannot be condensed and its English be improved.